nents of the voltages for the internal machine buses equal to

$$e_i'^{(2)} = |E_i'| \cos\left(\delta_{i(t)} + \frac{k_{2i}}{2}\right)$$

$$f_i'^{(2)} = |E_i'| \sin\left(\delta_{i(t)} + \frac{k_{2i}}{2}\right) \qquad i = 1, 2, \ldots, m$$

The fourth estimates are obtained from

$$k_{4i} = [(\omega_{i(t)} + l_{3i}) - 2\pi f]\,\Delta t$$

$$l_{4i} = \frac{\pi f}{H_i}(P_{mi} - P_{ei}^{(3)})\,\Delta t \qquad i = 1, 2, \ldots, m$$

where $P_{ei}^{(3)}$ are obtained from a third solution of the network equations with internal voltage angles equal to $\delta_{i(t)} + k_{3i}$ and voltage components equal to

$$e_i'^{(3)} = |E_i'| \cos(\delta_{i(t)} + k_{3i})$$
$$f_i'^{(3)} = |E_i'| \sin(\delta_{i(t)} + k_{3i})$$

The final estimates of the internal voltage angles and machine speeds at time $t + \Delta t$ are obtained by substituting the k's and l's into equations (10.5.3). The internal voltage angles $\delta_{i(t+\Delta t)}$ are used to calculate the estimates for the components of voltages for the internal machine buses from

$$e'_{i(t+\Delta t)} = |E_i'| \cos \delta_{i(t+\Delta t)}$$
$$f'_{i(t+\Delta t)} = |E_i'| \sin \delta_{i(t+\Delta t)} \qquad i = 1, 2, \ldots, m$$

The network equations are solved then for the fourth time to obtain bus voltages for the calculation of machine currents and powers and network power flows. The time is advanced by Δt and a network solution is obtained for any scheduled switching operation and change in the fault condition. The process is repeated until t equals the maximum time T_{max}.

10.6 Example of transient stability calculations

The method for determining transient stability will be illustrated with the sample power system used in Sec. 8.5 for the load flow problem. In this example the machines are represented by voltages of constant magnitudes behind direct-axis transient reactances. Loads are represented by fixed admittances to ground.

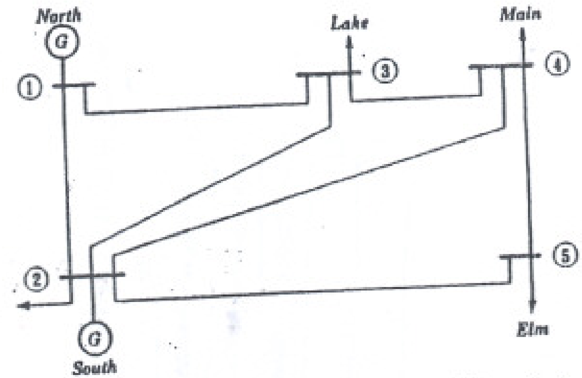

Fig. 10.8 Sample system for transient stability calculations.

Problem

Using the bus admittance matrix and the Gauss-Seidel iterative method for the solution of the network equations and the modified Euler method for the solution of the swing equations:

a. Determine the effects on the sample power system shown in Fig. 10.8 of a three-phase fault on bus 2 for a duration of 0.1 sec.

b. Determine the effects of the fault on bus 2 for a duration of 0.2 sec.

Solution

The results of the load flow calculation prior to the fault are given in Table 10.1. The inertia constants, direct-axis transient reactances, and equivalent admittances of the generators at buses 1 and 2 in per unit on a 100,000 kva base are given in Table 10.2.

Table 10.1 Bus voltages, generation, and loads from load flow calculation prior to fault

Bus code p	Bus voltages E_p	Generation		Load	
		Megawatts	Megavars	Megawatts	Megavars
1	1.06000 + j0.00000	129.565	−7.480	0.0	0.0
2	1.04621 − j0.05128	40.0	30.0	20.0	10.0
3	1.02032 − j0.08920	0.0	0.0	45.0	15.0
4	1.01917 − j0.09506	0.0	0.0	40.0	5.0
5	1.01209 − j0.10906	0.0	0.0	60.0	10.0

Table 10.2 Inertia constants, direct-axis transient reactances, and equivalent admittances for generators of sample system

Bus code p–i	Inertia constant H	Direct-axis transient reactance x'_d	Equivalent admittance y_{pi}
1–6	50.0	0.25	0.0 − j4.00000
2–7	1.0	1.50	0.0 − j0.66667

a. The Gauss-Seidel iterative equations describing the performance of the network, using the bus code numbers given in Fig. 10.9, are

$$E_1^{k+1} = -YL_{12}E_2^k - YL_{13}E_3^k - YL_{16}E_6$$
$$E_2^{k+1} = -YL_{21}E_1^{k+1} - YL_{23}E_3^k - YL_{24}E_4^k - YL_{25}E_5^k - YL_{27}E_7$$
$$E_3^{k+1} = -YL_{31}E_1^{k+1} - YL_{32}E_2^{k+1} - YL_{34}E_4^k$$
$$E_4^{k+1} = -YL_{42}E_2^{k+1} - YL_{43}E_3^{k+1} - YL_{45}E_5^k$$
$$E_5^{k+1} = -YL_{52}E_2^{k+1} - YL_{54}E_4^{k+1}$$

The line parameters YL_{pq} for these equations can be obtained from the elements of the bus admittance matrix used for the load flow solution prior to the disturbance and the equivalent admittances for machines and loads.

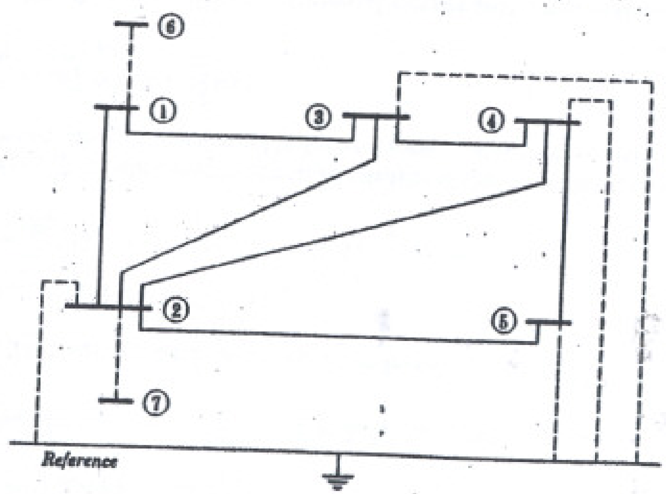

Fig. 10.9 Representation of sample system for transient stability calculations.

The bus admittance matrix is

Y_{BUS} =

	1	2	3	4	5
1	6.25000 − j18.69500	−5.00000 + j15.00000	−1.25000 + j3.75000		
2	−5.00000 + j15.00000	10.83334 − j32.41500	−1.66667 + j5.00000	−1.66667 + j5.00000	−2.50000 + j7.50000
3	−1.25000 + j3.75000	−1.66667 + j5.00000	12.91667 − j38.69500	−10.00000 + j30.00000	
4		−1.66667 + j5.00000	−10.00000 + j30.00000	12.91667 − j38.69500	−1.25000 + j3.75000
5		−2.50000 + j7.50000		−1.25000 + j3.75000	3.75000 − j11.21000

The line parameters are obtained from the equation

$$YL_{pq} = Y_{pq}L_p = Y_{pq}\left(\frac{1}{Y_{pp}}\right)$$

The modified line parameter for element 1–2 is

$$YL_{12} = Y_{12}\left(\frac{1}{Y_{11} + y_{16}}\right)$$

$$YL_{12} = \frac{-5.00000 + j15.00000}{6.25000 - j22.69500}$$
$$= -0.67074 - j0.03560$$

where Y_{11} and Y_{12} are elements in the bus admittance matrix and y_{16} is the equivalent admittance representing the machine at bus 1, which is given in Table 10.2. The remaining line parameters for bus 1 are obtained from the equations

$$YL_{13} = Y_{13}\left(\frac{1}{Y_{11} + y_{16}}\right)$$

$$YL_{16} = Y_{16}\left(\frac{1}{Y_{11} + y_{16}}\right)$$

where

$$Y_{16} = -y_{16}$$

The line parameter for element 2–1 is obtained from

$$YL_{21} = Y_{21}\left(\frac{1}{Y_{22} + y_{27} + y_{20}}\right)$$

where Y_{22} and Y_{21} are elements in the bus admittance matrix; y_{27} is the equivalent admittance representing the machine at bus 2 and y_{20} is the equivalent admittance to ground representing the load at bus 2. The equation for the load equivalent admittance is

$$y_{p0} = \frac{P_{Lp} - jQ_{Lp}}{e_p^2 + f_p^2}$$

and for bus 2

$$y_{20} = \frac{0.20 - j0.10}{(1.04621)^2 + (0.05128)^2}$$
$$= 0.18228 - j0.09114$$

where the bus voltage is obtained from the load flow solution and is given in Table 10.1. The line parameter YL_{21} is

$$YL_{21} = \frac{-5.00000 + j15.00000}{11.01562 - j33.17281}$$
$$= -0.45235 - j0.00052$$

The YL_{pq}'s for all elements are given in Table 10.3.

The voltages behind the equivalent admittances representing the machines are obtained from the equation

$$E_i' = E_{ti} + jx_{di}'I_{ti} \qquad i = n+1, n+2, \ldots, n+m$$

where

$$I_{ti} = \frac{P_{ti} - jQ_{ti}}{E_{ti}^*}$$

and n is the number of buses of the network and m is the number of machines. For the machine at bus 1

$$E_6 = 1.06 + j0.0 + j0.25\left(\frac{1.29565 + j0.07480}{1.06 - j0.0}\right)$$
$$= 1.04236 + j0.30558$$

Table 10.3 Line parameters for transient stability representation of sample system

Bus code p–q	YL_{pq}
1–2	−0.67074 − j0.03560
1–3	−0.16769 − j0.00890
1–6	−0.16383 + j0.04512
2–1	−0.45235 − j0.00052
2–3	−0.15078 − j0.00017
2–4	−0.15078 − j0.00017
2–5	−0.22618 − j0.00026
2–7	−0.01810 + j0.00601
3–1	−0.09625 + j0.00080
3–2	−0.12833 + j0.00119
3–4	−0.77000 + j0.00711
4–2	−0.12866 + j0.00115
4–3	−0.77108 + j0.00687
4–5	−0.09650 + j0.00086
5–2	−0.65236 + j0.02966
5–4	−0.32618 + j0.01433

where the bus voltage and generation are obtained from Table 10.1 and the machine reactance from Table 10.2. The voltage magnitude is

$$|E_6| = 1.08623$$

and the internal voltage angle is

$$\delta_6 = 16.339° \qquad \text{or} \qquad \delta_6 = 0.28517 \text{ rad}$$

The voltage behind the equivalent admittance representing the machine at bus 2 is obtained in a similar manner and is

$$E_7 = 1.50335 + j0.49981$$

The voltage magnitude is

$$|E_7| = 1.58426$$

and the internal voltage angle is

$$\delta_7 = 18.390° \qquad \text{or} \qquad \delta_7 = 0.32097 \text{ rad}$$

The fault at bus 2 is simulated by setting the voltage at this bus equal to zero. Then the network equations are solved to obtain system conditions at the instant the fault occurs. In this calculation the voltage at the faulted bus and the voltages behind the equivalent admittances representing the machines are held fixed. The calculated system voltages are given in Table 10.4.

The machine currents, with the fault, are calculated from the equation

$$I_{ii} = (E'_i - E_{ii})y_{pi}$$

Then

$$\begin{aligned} I_{61} &= [(1.04236 + j0.30558) - (0.19234 + j0.00330)](0.0 - j4.0) \\ &= 1.20912 - j3.40008 \end{aligned}$$

Table 10.4 Bus voltages of sample system at the instant the fault occurs

Bus code p	Bus voltage E_p
1	0.19234 + j0.00330
2	0.0 + j0.0
3	0.04707 − j0.00096
4	0.03758 − j0.00118
5	0.01226 − j0.00093

and

$$\begin{aligned} I_{72} &= [(1.50335 + j0.49981) - (0.0 + j0.0)](0.0 - j0.66667) \\ &= 0.33321 - j1.00223 \end{aligned}$$

The electrical power of the machines is calculated from

$$P_{ei} - jQ_{ei} = I_{ii}(E'_i)^*$$

The real power of the machine at bus 1 is

$$\begin{aligned} P_{e6} &= (1.20912)(1.04236) - (3.40008)(0.30558) \\ &= 0.22134 \end{aligned}$$

The real power of the machine at bus 2 is zero since bus 2 is the faulted bus and its voltage is zero. Calculating the real power as a check,

$$\begin{aligned} P_{e7} &= (0.33321)(1.50335) - (1.00223)(0.49981) \\ &= 0.0000067 \end{aligned}$$

The initial estimates of the internal voltage angles and speed of the machines at $t + \Delta t$ are obtained from the differential equations. The rate of change in speed of the machines is calculated from

$$\frac{d\omega_i}{dt} = \frac{\pi f}{H_i}(P_{mi} - P_{ei(t)})$$

Then, at $t = 0$ for the machine at bus 1,

$$\begin{aligned} \left.\frac{d\omega_6}{dt}\right|_{(0)} &= \frac{3.1416(60)}{50.0}(1.29565 - 0.22134) \\ &= 4.05006 \end{aligned}$$

Similarly, for the machine at bus 2,

$$\begin{aligned} \left.\frac{d\omega_7}{dt}\right|_{(0)} &= \frac{3.1416(60)}{1.0}(0.40000 - 0.0) \\ &= 75.3984 \end{aligned}$$

Next, the initial estimates of the speed of the machines at $t + \Delta t$ are calculated from

$$\omega^{(0)}_{i(t+\Delta t)} = \omega^{(1)}_{i(t)} + \left.\frac{d\omega_i}{dt}\right|_{(t)}\Delta t$$

where $\omega^{(1)}_{i(t)}$ at $t = 0$ is the rated speed and equal to $2\pi f$ and $\Delta t = 0.02$. Then, for the machine at bus 1,

$$\begin{aligned} \omega^{(0)}_{6(0.02)} &= 2(3.1416)60 + (4.05006)0.02 \\ &= 376.992 + 0.08100 \\ &= 377.07300 \end{aligned}$$

Similarly, for the machine at bus 2,

$$\begin{aligned}\omega^{(0)}_{7(0.02)} &= 2(3.1416)60 + (75.3984)0.02\\ &= 376.992 + 1.50797\\ &= 378.49907\end{aligned}$$

The rates of change of the internal voltage angles are calculated next from

$$\frac{d\delta_i}{dt} = \omega_{i(t)} - 2\pi f$$

Since $\omega_{i(t)}$ at $t = 0$ is equal to $2\pi f$, then for the machines

$$\left.\frac{d\delta_6}{dt}\right|_{(0)} = 0.0 \qquad \text{and} \qquad \left.\frac{d\delta_7}{dt}\right|_{(0)} = 0.0$$

The initial estimates of the internal voltage angles of the machines are calculated from

$$\delta^{(0)}_{i(t+\Delta t)} = \delta^{(1)}_{i(t)} + \left.\frac{d\delta_i}{dt}\right|_{(t)} \Delta t$$

Then, for the machines, the internal voltage angles in radians are

$$\begin{aligned}\delta^{(0)}_{6(0.02)} &= 0.28517\\ \delta^{(0)}_{7(0.02)} &= 0.32007\end{aligned}$$

The new components of the voltages behind the equivalent admittances representing the machines are calculated from

$$e'^{(0)}_{i(t+\Delta t)} = |E'_i| \cos \delta^{(0)}_{i(t+\Delta t)}$$

and

$$f'^{(0)}_{i(t+\Delta t)} = |E'_i| \sin \delta^{(0)}_{i(t+\Delta t)}$$

These voltage components replace the previous values obtained from the load flow solution prior to the fault and again the network equations are solved. In this calculation the new voltages behind the machine equivalent admittances as well as the zero voltage at the faulted bus are held constant.

Since there is no change in the internal voltage angle for the initial estimate, the system voltages and machine currents and powers are the same as those obtained from the network solution at the instant the fault occurs. Consequently, the rates of change in the speed of the machines at $t + \Delta t = 0.02$ will be the same. Therefore,

$$\left.\frac{d\omega_6}{dt}\right|_{(0.02)} = 4.05006 \qquad \text{and} \qquad \left.\frac{d\omega_7}{dt}\right|_{(0.02)} = 75.39484$$

The final estimate for the speed of the machines at $t + \Delta t$ is calculated from

$$\omega^{(1)}_{i(t+\Delta t)} = \omega^{(1)}_{i(t)} + \left(\frac{\left.\frac{d\omega_i}{dt}\right|_{(t)} + \left.\frac{d\omega_i}{dt}\right|_{(t+\Delta t)}}{2}\right)\Delta t$$

Then, for the machine at bus 1,

$$\begin{aligned}\omega^{(1)}_{6(0.02)} &= 2(3.1416)60 + \left(\frac{4.05006 + 4.05006}{2}\right)0.02\\ &= 377.07300\end{aligned}$$

Similarly, for the machine at bus 2,

$$\begin{aligned}\omega^{(1)}_{7(0.02)} &= 2(3.1416)60 + \left(\frac{75.3984 + 75.3984}{2}\right)0.02\\ &= 378.40997\end{aligned}$$

The rates of change of the internal voltage angles at $t + \Delta t$ are calculated from

$$\frac{d\delta_i}{dt} = \omega^{(1)}_{i(t+\Delta t)} - 2\pi f$$

Then, for the machine at bus 1,

$$\begin{aligned}\left.\frac{d\delta_6}{dt}\right|_{(.02)} &= 377.0730 - 376.9920\\ &= 0.08100\end{aligned}$$

Similarly, for the machine at bus 2,

$$\begin{aligned}\left.\frac{d\delta_7}{dt}\right|_{(0.02)} &= 378.49997 - 376.9920\\ &= 1.50797\end{aligned}$$

The final estimates for the internal voltage angles of the machines at $t + \Delta t$ are calculated from

$$\delta^{(1)}_{i(t+\Delta t)} = \delta^{(1)}_{i(t)} + \left(\frac{\left.\frac{d\delta_i}{dt}\right|_{(t)} + \left.\frac{d\delta_i}{dt}\right|_{(t+\Delta t)}}{2}\right)\Delta t$$

Then, for the machine at bus 1,

$$\begin{aligned}\delta^{(1)}_{6(0.02)} &= 0.28517 + \left(\frac{0.0 + 0.08100}{2}\right)0.02\\ &= 0.28517 + 0.00081\\ &= 0.28598\end{aligned}$$

Similarly, for the machine at bus 2,

$$\delta^{(1)}_{7(0.02)} = 0.32097 + \left(\frac{0.0 + 1.50797}{2}\right) 0.02$$
$$= 0.32097 + 0.01508$$
$$= 0.33605$$

The internal voltage angles in degrees at $t + \Delta t = 0.02$ are

$$\delta^{(1)}_{6(0.02)} = 0.28598 \left(\frac{180}{\pi}\right) = 16.38540°$$

and

$$\delta^{(1)}_{7(0.02)} = 0.33605 \left(\frac{180}{\pi}\right) = 19.25420°$$

At $t + \Delta t = 0.02$ the final components of voltages behind the machine equivalent admittances are

$$e'^{(1)}_6 = 1.08623 \cos(16.38540)$$
$$= 1.04212$$
$$f'^{(1)}_6 = 1.08623 \sin(16.38540)$$
$$= 0.30641$$

and

$$e'^{(1)}_7 = 1.58426 \cos(19.25420)$$
$$= 1.49564$$
$$f'^{(1)}_7 = 1.58426 \sin(19.25420)$$
$$= 0.52243$$

Then the network equations are solved to obtain the final system voltages at $t + \Delta t = 0.02$. The voltages obtained from this calculation are given in Table 10.5.

Table 10.5 Bus voltages of sample system at $t + \Delta t = 0.02$

Bus code p	Bus voltage E_p
1	$0.19258 + j0.00353$
2	$0.0 + j0.0$
3	$0.04815 - j0.00114$
4	$0.03845 - j0.00133$
5	$0.01249 - j0.00097$

With these system voltages the machine currents and powers at $t + \Delta t = 0.02$ can be calculated. The current of the machine at bus 1 is

$$I_{61(0.02)} = [(1.04212 + j0.30641) - (0.19258 + j0.00353)](0.0 - j4.0)$$
$$= 1.21152 - j3.39816$$

and the real power is

$$P_{e6(0.02)} = (1.21152)(1.04212) - (3.39816)(0.30641)$$
$$= 0.22132$$

The current of the machine at bus 2 is

$$I_{72(0.02)} = [(1.49564 + j0.52243) - (0.0 + j0.0)](0.0 - j0.66667)$$
$$= 0.34829 - j0.99710$$

and the real power is zero since the fault is at bus 2.

This completes the calculations for values at $t + \Delta t = 0.02$. Then the time is set to $t = 0.02$ and the process repeated to obtain estimates at $t + \Delta t = 0.04$. When the time is advanced to $t = 0.10$, however, the network equations are solved without the fault to obtain the post fault conditions before proceeding with the normal process. In the network calculation only the voltages behind the machine equivalent admittances are held constant. The machine currents and powers obtained with the new system voltages are used to obtain new estimates at $t + \Delta t = 0.12$. The process is continued until $t = T_{max}$.

The internal voltage angles and the ratios of actual to rated speed of the machines for the complete calculation are shown in Figs. 10.10 and 10.11, respectively. The system is stable for this disturbance.

b. The procedure for determining the transient stability of the sample system for a fault on bus 2 of duration 0.2 sec is identical except that the network solution without the fault is obtained when $t = 0.20$ instead of $t = 0.10$ as in part *a*. The internal voltage angles and the ratios of actual to rated speed of the machines for the complete calculation are shown in Figs. 10.12 and 10.13. The system is unstable for this disturbance.

10.7 *Exciter and governor control systems*

In the solution techniques described in Sec. 10.5 the effects of the exciter and governor control systems on power system response were neglected. In that representation the field voltage E_{fd} and the mechanical power P_m were held constant in the transient calculations. When a more detailed evaluation of system response is required or the period of analysis extends beyond one second it is important to include the effects of the exciter and governor systems.

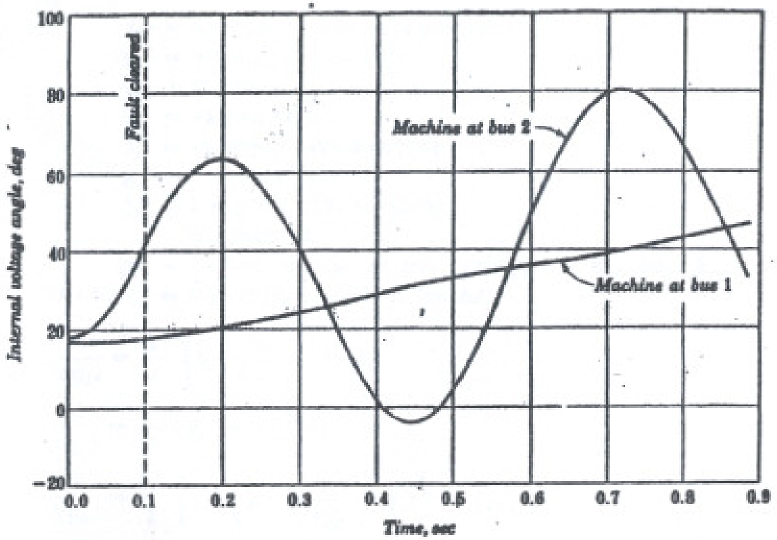

Fig. 10.10 *Internal voltage angle of machine with respect to time for a fault duration of 0.1 sec.*

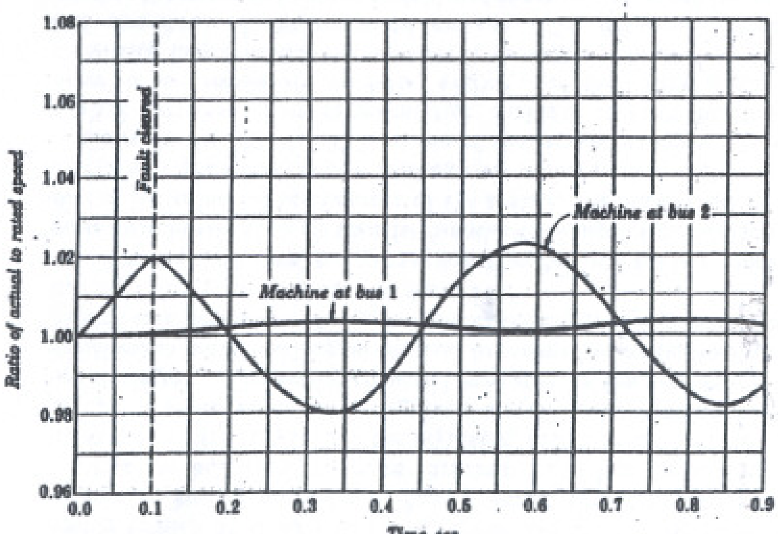

Fig. 10.11 *Ratio of actual to rated speed of machine with respect to time for a fault duration of 0.1 sec.*

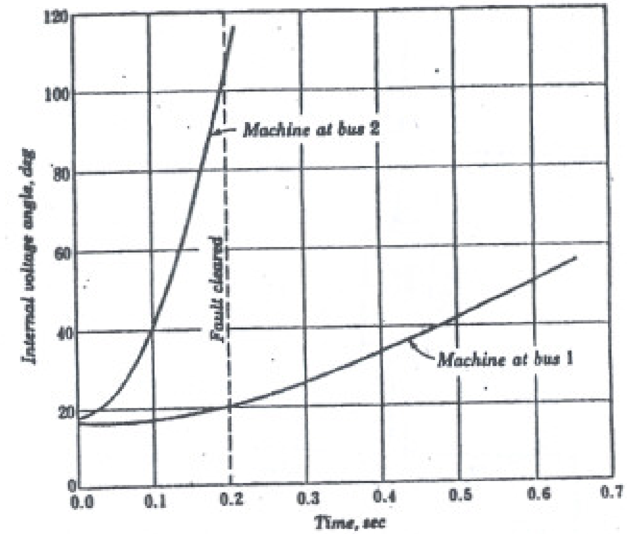

Fig. 10.12 *Internal voltage angle of machine with respect to time for a fault duration of 0.2 sec.*

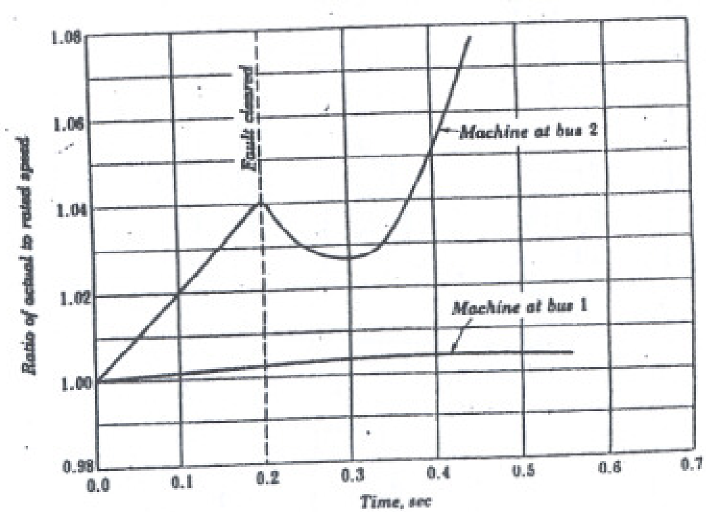

Fig. 10.13 *Ratio of actual to rated speed of machine with respect to time for a fault duration of 0.2 sec.*

The exciter control system provides the proper field voltage to maintain a desired system voltage, usually at the high-voltage bus of the power plant. An important characteristic of an exciter control system is its ability to respond rapidly to voltage deviations during both normal and emergency system operation. Many different types of exciter control systems are employed on power systems. The basic components of an exciter control system are the regulator, amplifier, and exciter. The regulator measures the actual regulated voltage and determines the voltage deviation. The deviation signal produced by the regulator is then amplified to provide the signal required to change the exciter field current. This in turn produces a change in the exciter output voltage which results in a new excitation level for the generator.

A convenient form of representing a control system is a block diagram that relates through transfer functions the input and output variables of the principal components of the system. A block diagram for a simplified representation of a continuously acting exciter control system is shown in Fig. 10.14. This is one of the important types of exciter control systems. This representation includes transfer functions to describe the regulator, amplifier, exciter, and stabilizing loop. The stabilizing loop modifies the response to eliminate undesired oscillations and overshoot of the regulated voltage. The differential equations relating the input and output variables of the regulator, amplifier, exciter, and stabilizing loop, respectively, are

$$\begin{aligned}
\frac{dE^{v}}{dt} &= \frac{1}{T_R}(E_S - E_t - E^{v}) \\
\frac{dE^{iii}}{dt} &= \frac{1}{T_A}\left\{K_A\left(E^{v} + \frac{E_0^{iii}}{K_A} - E^{iv}\right) - E^{iii}\right\} \\
\frac{dE_{fd}}{dt} &= \frac{1}{T_E}(E^{ii} - K_E E_{fd}) \\
\frac{dE^{iv}}{dt} &= \frac{1}{T_F}\left\{K_F\frac{dE_{fd}}{dt} - E^{iv}\right\}
\end{aligned} \tag{10.7.1}$$

where E_S = scheduled voltage in per unit
E_0^{iii} = output voltage of the amplifier in per unit prior to the disturbance
T_R = regulator time constant
K_A = amplifier gain
T_A = amplifier time constant
K_E = exciter gain
T_E = exciter time constant
K_F = stabilizing loop gain
T_F = stabilizing loop time constant

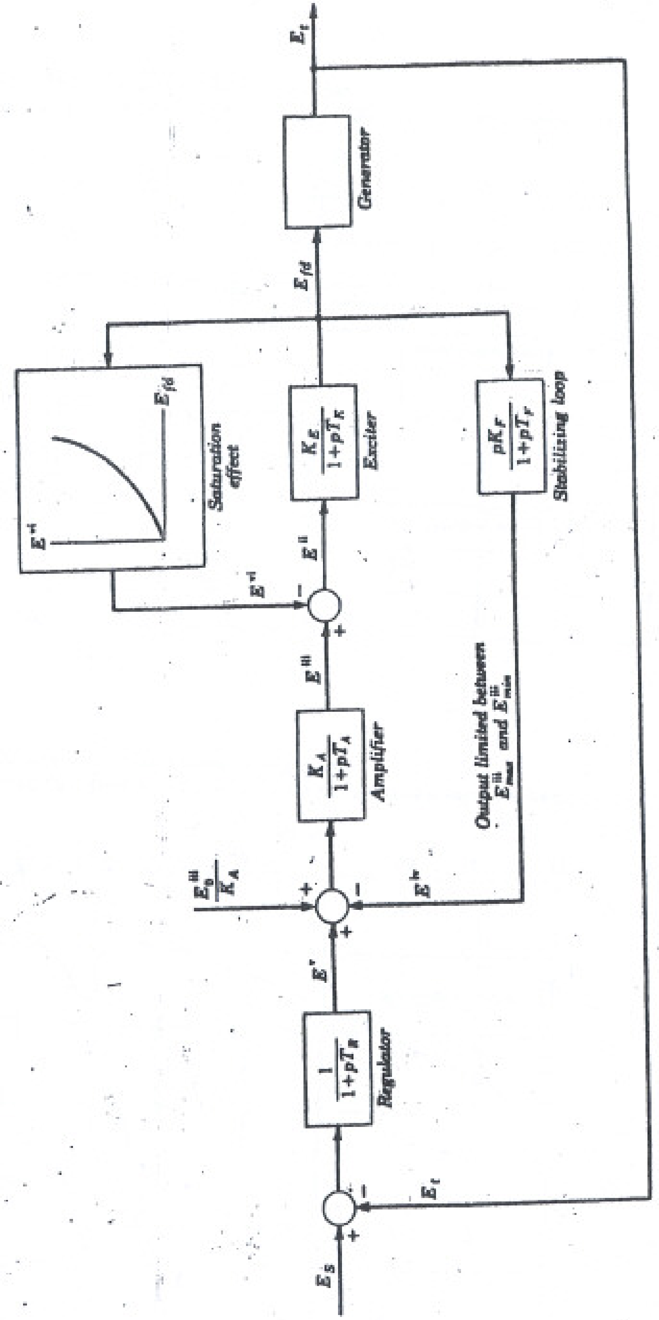

Fig. 10.14 Block diagram for a representation of an exciter control system.