1 Simulation Results

In this section, simulation results are presented to illustrate the performance of the proposed control scheme. The corresponding algorithm is presented in Table 1.

Table 1: Algorithm for Adaptive Visual Servoing without image velocity measurement

<table>
<thead>
<tr>
<th>Robotic System</th>
<th>( y_c = K_p y + y_{\alpha} ), ( y = k(q) ), ( \dot{y} = J(q) \dot{q} = W(q, \dot{q}) b ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressor ( M(q) \dot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau ). ( \Omega^T = \omega^T ), ( \Omega^T = [\omega^T \dot{\omega}_2] ).</td>
<td></td>
</tr>
<tr>
<td>Vector ( \omega = \frac{2\lambda_2 \lambda_0}{\lambda(s)} \ y_c - (\lambda_2^2 + 2\lambda_2 \lambda_0) y_c + \lambda_2^2 r ).</td>
<td></td>
</tr>
<tr>
<td>Filtered signals ( \psi_f = L^{-1}(s) \omega ), ( \dot{\psi}_f = \theta^T \Psi_1 ), ( L(s) = (s + \lambda_c) I ).</td>
<td></td>
</tr>
<tr>
<td>Output error ( e_c = y_c - y_{\alpha} ), ( y_{\alpha} = G_m(s) r ), ( G_m(s) = \lambda_2^2 / (s + \lambda_c)^2 I ).</td>
<td></td>
</tr>
<tr>
<td>Robot Control law ( \tau = Y(q, \dot{q}, \dot{q}_r, \dot{q}_A) \ \dot{\alpha} - K_D \sigma ), ( K_D = K_D^T &gt; 0 ).</td>
<td></td>
</tr>
<tr>
<td>Cascade Strategy ( \dot{q}_r = \dot{J}^{-1}(q) H^{-1}(s) [v + \lambda \dot{J}(q) \dot{q}] ), ( H(s) = (s + \lambda) I ).</td>
<td></td>
</tr>
<tr>
<td>Visual Servoing law ( v_i = \dot{v}_i - 2\lambda_c \Lambda^{-1}(s) v_i ), ( \dot{v}_i = \hat{\theta}^T \Psi_1 + \theta^T \Omega_i ), ( \Lambda(s) = (s + \lambda_0) I ).</td>
<td></td>
</tr>
<tr>
<td>Adaptation laws ( \dot{\alpha} = -\Gamma_d Y^T \sigma ), ( \Gamma_d = \Gamma_d^T &gt; 0 ).</td>
<td></td>
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<tr>
<td>( \dot{b} = -\Gamma_k W^T \epsilon ), ( \Gamma_k = \Gamma_k^T &gt; 0 ).</td>
<td></td>
</tr>
<tr>
<td>( \dot{\gamma}<em>i = -\gamma_i e</em>{ci} \psi_i ), ( \gamma_i &gt; 0 ).</td>
<td></td>
</tr>
</tbody>
</table>

Here, we consider the visual servoing model (2), as well as the robot kinematic and dynamic models (3) and (4) respectively. Uncertainty of the camera and robot parameters is compensated by using the adaptive control approach developed in this paper. We consider that a two-link manipulator is moving on a horizontal plane with forward kinematics map \( y = k(q) \) given by

\[
\begin{align*}
y_1 &= l_1 \cos(q_1) + l_2 \cos(q_1 + q_2), \\
y_2 &= l_1 \sin(q_1) + l_2 \sin(q_1 + q_2),
\end{align*}
\]

where \( q = [q_1, q_2]^T \) is the joint angle vector of the robot, \( l_1, l_2 \) stands for link lengths, and the elements of the analytical Jacobian \( J(q) \) are: \( J_{11} = -b_1 \sin(q_1) - b_2 \sin(q_1 + q_2) \), \( J_{12} = -b_2 \sin(q_1 + q_2) \), \( J_{21} = b_1 \cos(q_1) + b_2 \cos(q_1 + q_2) \), \( J_{22} = b_2 \cos(q_1 + q_2) \), with \( b_1 = l_1 \), \( b_2 = l_2 \).

On the other hand, we have that the components of matrices \( M(q), C(q, \dot{q}) \) and \( g(q) \) are: \( M_{11} = a_1 + 2a_2 \cos(q_2) \), \( M_{12} = M_{21} = a_3 + a_2 \cos(q_2) \), \( M_{22} = a_3 \), \( h_2 = a_2 \sin(q_2) \), \( C_{11} = -h_2 q_2 \), \( C_{12} = -h_2 (q_1 + q_2) \), \( C_{21} = h_2 q_1 \), \( C_{22} = 0 \), \( q_1 = a_3 g \sin(q_1) + a_5 g \sin(q_1 + q_2) \), \( q_2 = a_5 g \sin(q_1 + q_2) \), with \( a_1 = l_1 + m_1 l_1^2 + l_2 + m_2 l_2^2 + m_1 l_1^2, a_2 = m_2 l_1 l_2, a_3 = l_2 + m_2 l_2^2, a_4 = m_1 l_1^2 + l_2 l_1, a_5 = m_2 l_2. \)
The parameter values were chosen to be the ones in [1], say: $m_1 = 9.5$ kg; $l_1 = 0.20$ m; $l_{c1} = 0.12$ m; $I_1 = 4.3 \times 10^{-3}$ kg m$^2$; $m_2 = 4.5$ kg; $I_2 = 6.1 \times 10^{-3}$ kg m$^2$; $l_2 = 0.08$ m; $l_{c2} = 0.08$ m; $g = 9.8$ m s$^{-2}$.

The desired trajectory $y_{cd}$ was designed to be the output of the model reference $G_m(s) = \frac{100}{s + 10}$ in response to the external reference signals

\begin{align*}
    r_1 &= c_1 \sin(w_r t) + c_2 + c_4 \sin(1.5 w_r t), \quad (3) \\
    r_2 &= c_1 \sin(w_r t + c_5) + c_3 + c_4 \sin(1.5 w_r t + c_5), \quad (4)
\end{align*}

where $w_r = 1$ rad s$^{-1}$, $c_5 = 1.6$ rad, $c_1 = c_4 = 50$ and $c_2 = c_3 = 300$. The parameters used in the simulations were: $K_D = \text{diag}(200, 20)$; $\lambda_c = 10$; $\Gamma_d = 20 I$; $\lambda_p = 1$; $\Gamma_k = 10 I$; $\gamma_1 = \gamma_2 = 10$; $\lambda = 10$; $\lambda_0 = 10$; $\phi = -\frac{\pi}{6}$ rad; $f = 0.008$ m, $z_0 = 0.64$ m; $a_1 = a_2 = 72727$ pixel m$^{-1}$. The initial conditions are $q(0) = [-\frac{\pi}{20} \frac{\pi}{2}]^T$, $y_{cd}(0) = [250 \ 450]^T$, $\theta_1(0) = [10^{-3} \ 0 \ 0]^T$; $\theta_2(0) = [0 \ 10^{-3}]^T$, $\hat{a}(0) = 0.9 [a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T$ and $\hat{b}(0) = 0.9 [b_1 \ b_2]^T$. All other initial conditions are null. The initial values of the parameters $\theta_1(0)$ and $\theta_2(0)$ were chosen from the best tuning for the non-adaptive case with $\phi \approx 0$ rad.

Simulation results, obtained from MATLAB/Simulink (The MathWorks Inc.), are presented in Figures 1-4. The time history of the image error and angular position error are shown in Figures 1(a) and 1(b) respectively, where it can be observed the asymptotic convergence of the image error to a small residual set of 2 pixel. Figures 2(a) and 2(b) depict the convergence of dynamic and kinematic adaptive parameters. The convergence of the camera adaptive parameters is illustrated in Figures 2(c) and 2(d). The time history of the cartesian control signal and joint torque signal is shown in Figures 3(a) and 3(b) respectively. The trajectory tracking in the image space and the operational space is depicted in Figures 4(a) and 4(b), where it can be observed that, in despite of the transient behavior and the evident misalignment of the camera, a good performance was achieved.

![Figure 1: Errors: (a) image tracking, (b) angular position](image-url)
Figure 2: Adaptive parameters: (a) dynamic $\hat{a}$, (b) kinematic $\hat{b}$, (c) camera $\hat{\theta}_1$, (d) camera $\hat{\theta}_2$.

Figure 3: Control signals: (a) cartesian control, (b) joint torques.
Figure 4: Trajectory tracking: (a) image space, (b) Cartesian space.

References