Authors: Jonathan , Antonio C. Leite and Fernando Lizarralde
Title: Uncalibrated Image-Based Visual Servoing Approach for Translational Trajectory Tracking with an Uncertain Robot Manipulator
Address: Dept. of Electrical Eng., COPPE, Federal University of Rio de Janeiro, Brazil.
Faculty of Science and Techonology, Norwegian University of Life Sciences, Norway

1 Parameterization of the Image Jacobian

The parameterization of the Image Jacobian is presented for a pinhole camera and a 7DOF collaborative robot manipulator Kinova Gen3. The proposed adaptive visual servoing algorithm is presented in Table 1.

Table 1: Algorithm for Adaptive Visual Servoing without image velocity measurement

Robotic System	$\dot{p}_v = J_v(\theta, p_v)\,\dot{\theta} \qquad p_v = [p_{xy}^T\ a_v]^T$ $M(\theta)\,\ddot{\theta} + C(\theta,\dot{\theta})\,\dot{\theta} + G(\theta) = Y_d(\theta,\dot{\theta},\dot{\theta},\ddot{\theta})\ a_d = \tau$
Parameterization	$\hat{J}_\perp(\theta)\,w = Y_\perp(\theta,w)\ \hat{a}_\perp$ $\hat{J}_z(\theta)\,w = Y_z(\theta,w)\ \hat{a}_z$ $\hat{J}_v(\theta,p_v)\ w = \bar{Y}_\perp(\theta,p_v,w)\ \hat{a}_\perp - \bar{Y}_z(\theta,p_v,w)\ \hat{a}_z$
Regressor Matrices	$\bar{Y}_\perp(\theta,p_v,w) = \begin{bmatrix} (a_v)^{\frac{1}{2}} I_2 \\ 0 \quad 0 \end{bmatrix} Y_\perp(\theta,w)$ $\bar{Y}_z(\theta,p_v,w) = \begin{bmatrix} (a_v)^{\frac{1}{2}}(p_{xy}-o_c) \\ 2(a_v)^{\frac{3}{2}} \end{bmatrix} Y_z(\theta,w)$
Output error	$e_v = p_v - p_{vd} \qquad e_o = \hat{p}_{xy} - p_{xy} \qquad e_{xy} = p_{xy} - p_{xyd}$ $e_a = a_v - a_{vd} \qquad p_{vd} = [p_{xyd}^T\ a_{vd}]^T$
Robot Control law	$\tau = Y_d(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r)\ \hat{a}_d - K_D\,\sigma\,, \qquad K_D = K_D^T > 0\,.$ $\sigma = \dot{\theta} - \dot{\theta}_r$
Cascade Strategy	$\dot{\theta}_r = u$
Visual Servoing law	$u = \hat{J}_v^{\dagger}(\theta,\hat{p}_v)\ [\dot{p}_{vd} - K_v\,(\hat{p}_v - p_{vd})] \qquad K_v = \text{diag}\{K_{xy}, k_a\}$
Adaptive Observer	$\dot{\hat{p}}_{xy} = (a_v)^{\frac{1}{2}} \left[\hat{J}_\perp - (p_{xy} - o_c + e_{xy})\,\hat{J}_z\right] u - K_o e_o + K_{xy}\,e_{xy}$ $\hat{p}_v = [\hat{p}_{xy}^T \quad a_v]^T \qquad K_o = K_o^T > 0 \quad K_{xy} = K_{xy}^T > 0$
Adaptation laws	$\dot{\hat{a}}_d = -\Gamma_d\,Y_d^T(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r)\,\sigma\,, \qquad \Gamma_d = \Gamma_d^T > 0\,.$ $\dot{\hat{a}}_\perp = \Gamma_\perp\,\bar{Y}_\perp^{\mathsf{T}}(\theta,p_v,u) \begin{bmatrix} e_{xy} - e_o \\ e_a \end{bmatrix} \qquad \Gamma_\perp = \Gamma_\perp^T > 0\,.$ $\dot{\hat{a}}_z = -\Gamma_z\,\bar{Y}_z^{\mathsf{T}}(\theta,p_v,u) \begin{bmatrix} e_{xy} - e_o \\ e_a \end{bmatrix} \qquad \Gamma_z = \Gamma_z^T > 0\,.$

1.1 Kinova Gen3 Kinematics

Here the Kinova Gen3 manipulator (Figure 1) is considered.

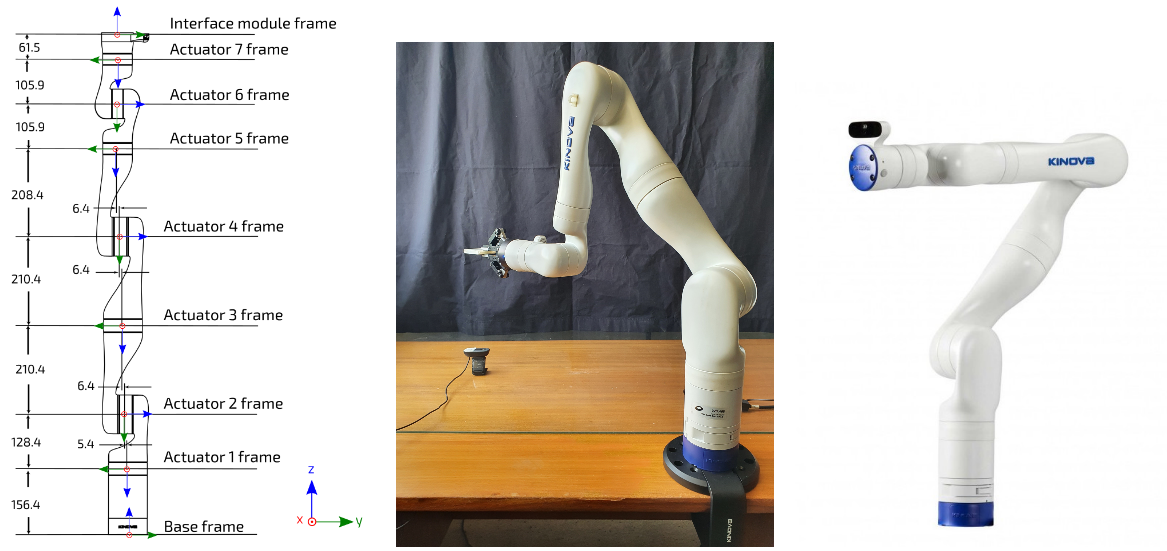

Figure 1: Schematics of the Kinova Gen3 7-DoF manipulator

The Denavit-Hartenberg parameters for this manipulator are presented in table 2.

Link	α (rad)	a (m)	θ (rad)	d (m)
0 (from base)	π	0	0	0.285
1	$\pi/2$	0	θ_1	0
2	$\pi/2$	0	$\theta_2+\pi$	0
3	$\pi/2$	0	$\theta_3+\pi$	-0.42
4	$\pi/2$	0	$-\theta_4+\pi$	0
5	$\pi/2$	0	$\theta_5+\pi$	-0.31
6	$\pi/2$	0	$-\theta_6+\pi$	0
7	π	0	$\theta_7+\pi$	-0.36

Table 2: Denavit-Hartenberg standard parameters for Kinova Gen3. Parameter d_7 includes the distance to the QR-code centroid.

Here, the Kinova Gen3 manipulator is controlled with 4-DoF, through joints θ_1, θ_2, θ_4, and θ_6. The remaining joints are considered fixed at $\theta_3=\pi$, $\theta_5=0$, $\theta_7=0$.

Consider the following abbreviations: $s_1=\sin(\theta_1)$, $c_1=\cos(\theta_1)$, $s_2=\sin(\theta_2)$, $c_2=\cos(\theta_2)$, $s_{24}=\sin(\theta_2-\theta_4)$, $c_{24}=\cos(\theta_2-\theta_4)$, $s_{246}=\sin(\theta_2-\theta_4-\theta_6)$, and $c_{246}=\cos(\theta_2-\theta_4-\theta_6)$.

Then, the forward kinematics is given by:

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ \phi \end{bmatrix} = \begin{bmatrix} c_1\ (l_2\ s_2 + l_3\ s_{24} + l_4\ s_{246}) \\ s_1\ (l_2\ s_2 + l_3\ s_{24} + l_4\ s_{246}) \\ l_1 + l_2\ c_2 + l_3\ c_{24} + l_4\ c_{246}) \\ \theta_2 - \theta_4 - \theta_6 \end{bmatrix}$$

where ϕ is the end-effector pitch, $l_1 = -d_1$, $l_2 = -d_3$, $l_3 = -d_5$, and $l_4 = -d_7$ are the manipulator links (d_i from table 2).

The manipulator differential kinematic is given by:

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \\ \dot{\phi} \end{bmatrix} = J(\theta)\ \dot{\theta}$$

where J is the manipulator Jacobian:

$$J(\theta) = \begin{bmatrix} -s_1(l_2 s_2 + l_3 s_{24} + l_4 s_{246}) & c_1(l_2 c_2 + l_3 c_{24} + l_4 c_{246}) & -c_1(l_3 c_{24} + l_4 c_{246}) & -c_1 l_4 c_{246} \\ -c_1(l_2 s_2 + l_3 s_{24} + l_4 s_{246}) & -s_1(l_2 c_2 + l_3 c_{24} + l_4 c_{246}) & s_1(l_3 c_{24} + l_4 c_{246}) & s_1 l_4 c_{246} \\ 0 & -(l_2 s_2 + l_3 s_{24} + l_4 s_{246}) & l_3 s_{24} + l_4 s_{246} & l_4 s_{246} \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

1.2 Kinematics Model

First, consider the following equation for the differential kinematics of position p_v of the target as seen through a pinhole camera, where the target is attached to the end-effector:

$$\dot{p}_v = \begin{bmatrix} \dot{p}_{xy} \\ \dot{a}_v \end{bmatrix} = \begin{bmatrix} (a_v)^{\frac{1}{2}} \left(J_\perp - (\, p_{xy} - o_c\,)\, J_z\,\right) \\ -2\,(a_v)^{\frac{3}{2}}\, J_z \end{bmatrix} \dot{\theta} = J_v(\theta, p_v)\ \dot{\theta}\,, \tag{1}$$

where $J_v(\theta, p_v)$ is the image feature Jacobian matrix, with $J_\perp = \beta K_{p_\perp} J(\theta)$ and $J_z = \beta K_{p_z} J(\theta)$ being the image plane and depth Jacobian matrices, respectively; $J(\theta)$ is the position Jacobian, $o_c = [320, 240]^T$, $\beta \in \mathbb{R}^+$ is the depth-to-area transformation constant given by,

$$\sqrt{a_v(t)}\ z_c(t) = \frac{1}{\beta} \qquad \forall t\ , \tag{2}$$

and, $K_{p_\perp}$ and K_{p_z} are given by

$$K_{p_\perp} = \begin{bmatrix} f\,\alpha & 0 & 0 \\ 0 & f\,\alpha & 0 \end{bmatrix} R_{bc}\,, \qquad K_{p_z} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} R_{bc}\,, \tag{3}$$

where $f > 0$ is the focal length of the camera lens $\alpha \in \mathbb{R}$ is the camera scaling factor in *pixels* per unit of length. $R_{bc} \in SO(3)$ is the rotation matrix of the camera frame with respect to the base frame.

In the following sections, we present the planar and depth parameterizations of the experimental setup.

1.3 Parameterization of the Planar Jacobian $J_\perp$

Considering the parameter vector $a_\perp$ given by:

$$a_\perp = \begin{bmatrix} a_{\perp_1} \\ a_{\perp_2} \\ a_{\perp_3} \\ a_{\perp_4} \\ a_{\perp_5} \\ a_{\perp_6} \end{bmatrix} = \begin{bmatrix} l_2\,\beta\,f\,\alpha\,\cos(\phi) \\ l_3\,\beta\,f\,\alpha\,\cos(\phi) \\ l_4\,\beta\,f\,\alpha\,\cos(\phi) \\ l_2\,\beta\,f\,\alpha\,\sin(\phi) \\ l_3\,\beta\,f\,\alpha\,\sin(\phi) \\ l_4\,\beta\,f\,\alpha\,\sin(\phi) \end{bmatrix},$$

the Planar Jacobian $J_\perp = \beta K_{p\perp} J(\theta)$ is given by:

$$J_\perp = \begin{bmatrix} J_{\perp_{11}} & J_{\perp_{12}} & J_{\perp_{13}} & J_{\perp_{14}} \\ J_{\perp_{21}} & J_{\perp_{22}} & J_{\perp_{23}} & J_{\perp_{24}} \end{bmatrix},$$

where

$$\begin{aligned}
J_{\perp_{11}} &= -a_{\perp_1} s_1 s_2 - a_{\perp_2} s_1 s_{24} - a_{\perp_3} s_1 s_{246} - a_{\perp_4} c_1 s_2 - a_{\perp_5} c_1 s_{24} - a_{\perp_6} c_1 s_{246}, \\
J_{\perp_{12}} &= a_{\perp_1} c_1 c_2 + a_{\perp_2} c_1 c_{24} + a_{\perp_3} c_1 c_{246} - a_{\perp_4} s_1 c_2 - a_{\perp_5} s_1 c_{24} - a_{\perp_6} s_1 c_{246}, \\
J_{\perp_{13}} &= -a_{\perp_2} c_1 c_{24} - a_{\perp_3} c_1 c_{246} + a_{\perp_5} s_1 c_{24} + a_{\perp_6} s_1 c_{246}, \\
J_{\perp_{14}} &= -a_{\perp_3} c_1 c_{246} + a_{\perp_6} s_1 c_{246}, \\
J_{\perp_{21}} &= a_{\perp_4} s_1 s_2 + a_{\perp_5} s_1 s_{24} + a_{\perp_6} s_1 s_{246} - a_{\perp_1} c_1 s_2 - a_{\perp_2} c_1 s_{24} - a_{\perp_3} c_1 s_{246}, \\
J_{\perp_{22}} &= -a_{\perp_4} c_1 c_2 - a_{\perp_5} c_1 c_{24} - a_{\perp_6} c_1 c_{246} - a_{\perp_1} s_1 c_2 - a_{\perp_2} s_1 c_{24} - a_{\perp_3} s_1 c_{246}, \\
J_{\perp_{23}} &= a_{\perp_5} c_1 c_{24} + a_{\perp_6} c_1 c_{246} + a_{\perp_2} s_1 c_{24} + a_{\perp_3} s_1 c_{246}, \\
J_{\perp_{24}} &= a_{\perp_6} c_1 c_{246} + a_{\perp_3} s_1 c_{246}.
\end{aligned}$$

Then, considering the control signal $u = [u_1\ u_2\ u_4\ u_6]^\mathsf{T}$, we can parameterize

$$J_\perp(\theta)\ u = Y_\perp(\theta, u)\ a_\perp$$

where

$$Y_\perp = \begin{bmatrix} Y_{\perp_{11}} & Y_{\perp_{12}} & Y_{\perp_{13}} & Y_{\perp_{14}} & Y_{\perp_{15}} & Y_{\perp_{16}} \\ Y_{\perp_{21}} & Y_{\perp_{22}} & Y_{\perp_{23}} & Y_{\perp_{24}} & Y_{\perp_{25}} & Y_{\perp_{26}} \end{bmatrix},$$

and

$$\begin{aligned}
Y_{\perp_{11}} &= -s_2 s_1 u_1 + c_2 c_1 u_2, \\
Y_{\perp_{12}} &= -s_{24} s_1 u_1 + c_{24} c_1 (u_2 - u_4), \\
Y_{\perp_{13}} &= -s_{246} s_1 u_1 + c_{246} c_1 (u_2 - u_4 - u_6), \\
Y_{\perp_{14}} &= -s_2 c_1 u_1 - s_1 c_2 u_2, \\
Y_{\perp_{15}} &= -s_{24} c_1 u_1 - c_{24} s_1 (u_2 - u_4), \\
Y_{\perp_{16}} &= -s_{246} c_1 u_1 - s_1 c_{246} (u_2 - u_4 - u_6), \\
Y_{\perp_{21}} &= -c_1 s_2 u_1 - c_2 s_1 u_2, \\
Y_{\perp_{22}} &= -s_{24} c_1 u_1 - c_{24} s_1 (u_2 - u_4), \\
Y_{\perp_{23}} &= -s_{246} c_1 u_1 - c_{246} s_1 (u_2 - u_4 - u_6), \\
Y_{\perp_{24}} &= s_2 s_1 u_1 - c_2 c_1 u_2, \\
Y_{\perp_{25}} &= s_{24} s_1 u_1 - c_{24} c_1 (u_2 - u_4), \\
Y_{\perp_{26}} &= s_{246} s_1 u_1 - c_{246} c_1 (u_2 - u_4 - u_6).
\end{aligned}$$

1.4 Parameterization of the Depth Jacobian J_z

Consider the parameter vector a_z given by

$$a_z = \begin{bmatrix} a_{z_1} \\ a_{z_2} \\ a_{z_3} \end{bmatrix} = \begin{bmatrix} l_2\,\beta \\ l_3\,\beta \\ l_4\,\beta \end{bmatrix}.$$

The depth Jacobian $J_z = \beta K_{pz} J(\theta)$ is given by:

$$J_z = \left[\begin{array}{cccc} J_{z_1} & J_{z_2} & J_{z_3} & J_{z_4} \end{array} \right],$$

where

$$\begin{aligned} J_{z_1} &= 0, \\ J_{z_2} &= -a_{z_1} s_2 - a_{z_2} s_{24} - a_{z_3} s_{246}, \\ J_{z_3} &= a_{z_2} s_{24} + a_{z_3} s_{246}, \\ J_{z_4} &= a_{z_3} s_{246}. \end{aligned}$$

Then, considering the control signal $u = [u_1 \; u_2 \; u_4 \; u_6]^\mathsf{T}$, we can parameterize

$$J_z(\theta) \; u = Y_z(\theta, u) \; a_z$$

where

$$Y_z = \left[\begin{array}{ccc} -s_2 u_2 & -s_{24}(u_2 - u_4) & -s_{246}(u_2 - u_4 - u_6) \end{array} \right].$$