National University of Singapore

EE2022 Electrical Energy Systems

Lecture 12: Transformer and Per Unit Analysis
27-02-2012

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Detailed Syllabus

20/01/2012	**Three-phase circuit analysis**: Introduction to three-phase circuit. Balanced three-phase systems.
20/01/2012	**Three-phase circuit analysis**: Delta-Wye connection, Relationship between phase and line quantities
27/01/2012	**Three-phase circuit analysis**: Per-phase analysis, Three-phase power calculation
27/01/2012	Guest Lecture "Energy & Environment, Smart Grid & Challenges Ahead" by Prof. J Nanda (IIT Delhi, IEEE Fellow)
30/01/2012	**Generator modeling**: Simple generator concept
03/02/2012	**Generator modeling**: Equivalent circuit of synchronous generators
03/02/2012	**Generator modeling**: Operating consideration of synchronous generators, i.e. Excitation voltage control, real power control, and loading capability
06/02/2012	**Generator modeling**: Principle of asynchronous generators
10/02/2012	**Transmission line modeling**: Overhead VS Underground cable
10/02/2012	**Transmission line modeling**: Four basic parameters of transmission line
13/02/2012	**Transmission line modeling**: Long transmission line model, Medium-length transmission line model, Short transmission line model
17/02/2012	**Transmission line modeling**: Operating consideration of transmission lines i.e. voltage regulation, line loadability, efficiency
17/02/2012	**Transformer and per unit analysis**: Principle of transformer, Single-phase transformer
27/02/2012	**Transformer and per unit analysis**: Single-phase per unit analysis
02/03/2012	**Transformer and per unit analysis**: Three-phase transformer, Three-phase per unit analysis
02/03/2012	**Review :** if time permits.

Learning Outcome

- Analyze three-phase balanced circuits.
  - Per-unit analysis for single-phase circuit.

Per Unit Quantity

Base Value

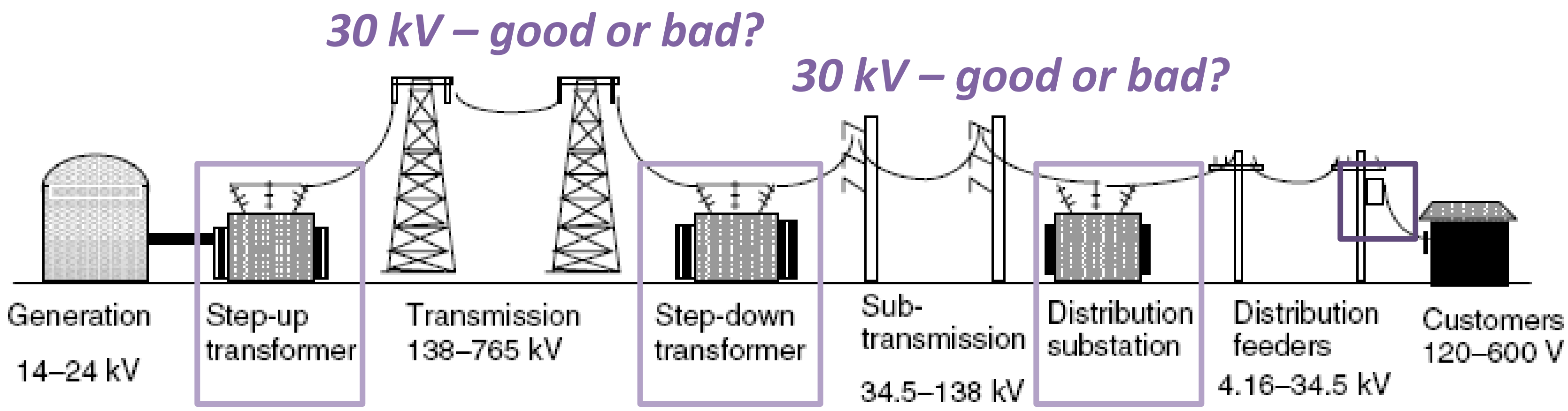
Change of Base

Steps of Calculation

SINGLE PHASE PER UNIT ANALYSIS

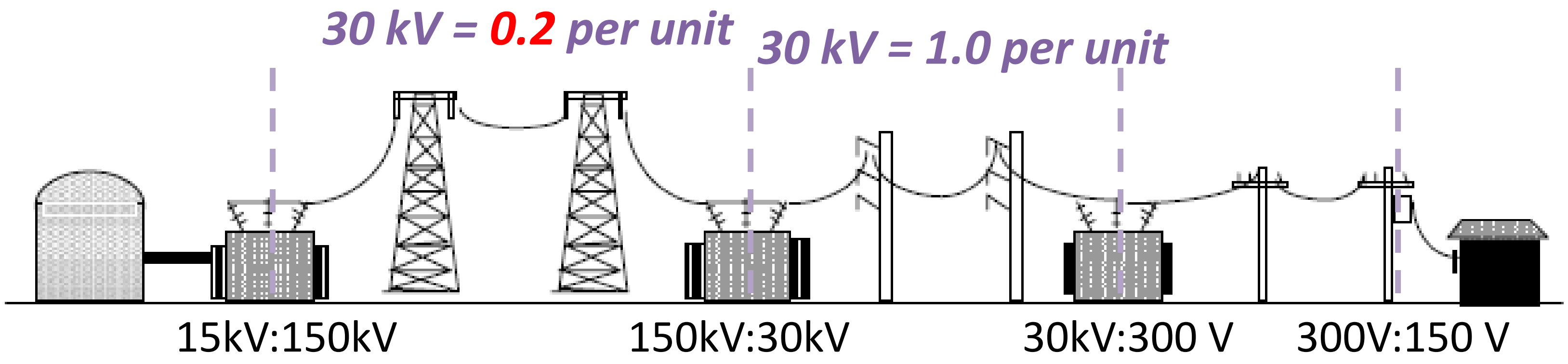
Motivations

- Transformer introduces various voltage levels.
- So far we can only reflect the load from one side of the transformer to another. Still we need to consider different voltage level at each side of the transformer when we try to find voltage and current.
- It is difficult to calculate voltage and current of the system at various points.
- It is even more difficult for the system operator to observe the current situation of the system.



Per Unit System

- Per unit system is when we ***normalize*** the voltage and current at each location.
- The normalization typically follows transformer ratings.
- This makes the per unit value of either voltage or current to be around 1.0 per unit.
- Per unit system allows the system operator to overlook abnormalities in the system easily.



Per Unit Quantity

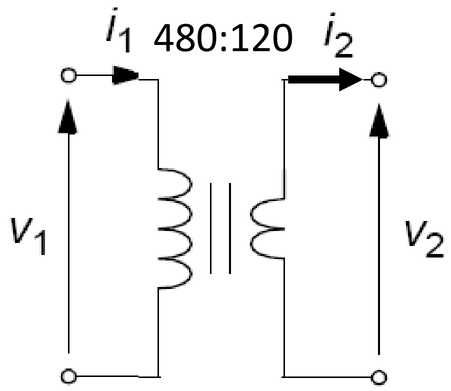
- The per unit quantity of voltage, current, power and impedance is found from dividing the actual quantity by a base value of that quantity.

$$\text{per} - \text{unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

- Per unit value is denoted by ‘p.u.’.
- The base value quantity typically follows transformer rating.
- The per unit values are the same irrespective of the sides of the transformer.

Example 1

- Consider a single phase 480/120 V transformer.
  - Choose the base value of voltage on the primary side to be 480 and that of secondary side to be 120.
  - If the voltage at primary is measured to be 432 V, which is 0.9 per unit, the voltage at secondary side is 108 V.
  - What is the per unit quantity on the secondary side?



$$\text{per}-\text{unit quantity} = \frac{108}{120} = 0.9$$

Base Value Selection

- Base values are **real numbers**, denote by subscript '*B*'.
- Voltage base values follow transformer voltage ratings.
- Only **single** base complex power $S_B^{1\Phi}$ in the system.
- The base value of power is used to normalize the quantity. Thus, the base values of real power, reactive power, and complex power are all the same real number.

$$P_B^{1\Phi} = Q_B^{1\Phi} = S_B^{1\Phi}$$

- Current base values are calculated from the base power and base voltage.
- Impedance base values (same value for impedance, resistance, or reactance) are calculated from voltage and current.

$$I_B = \frac{S_B^{1\Phi}}{V_B} \qquad Z_B = R_B = X_B = \frac{V_B}{I_B} = \frac{(V_B)^2}{S_B^{1\Phi}}$$

KVL, KCL, Complex Power Calculation

- We can still apply KVL, KCL, complex power calculation to the per unit value.
- The actual quantity is simply found from multiplying the per unit quantity (normalized quantity) with the base value.

$$S_B = V_B I_B$$

$$V_B = Z_B I_B$$

Think of Base value as ‘Normalization’.

$$S_{\text{p.u.}} = V_{\text{p.u.}} I_{\text{p.u.}}^*$$

$$V_{\text{p.u.}} = Z_{\text{p.u.}} I_{\text{p.u.}}$$

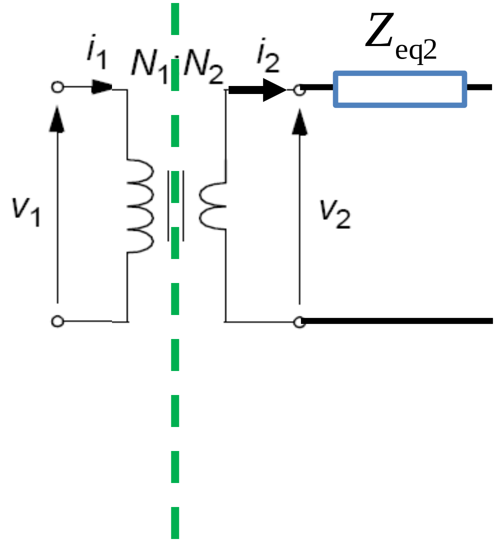
Example 2: Per Unit Value

- A single-phase 20kVA, 480/120 V, 60 Hz transformer has an equivalent leakage impedance referred to 120-volt winding of $Z_{\mathrm{eq2}} = 0.0525\angle 78.13°\ \Omega$. Using the transformer rating as base values, find per-unit leakage impedance.

$$S_B = 20\ \mathrm{kVA}$$

$$V_{B1} = 480\ \mathrm{V}$$

$$Z_{B1} = \frac{(V_{B1})^2}{S_B^{1\Phi}} = 11.52\ \Omega$$



$$V_{B2} = 120\ \mathrm{V}$$

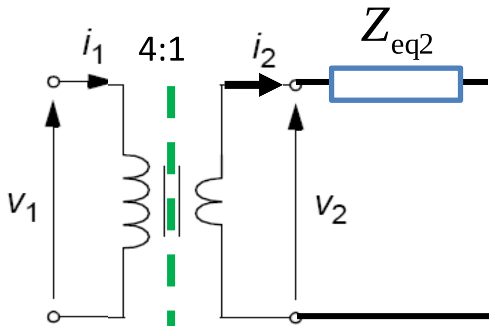
$$Z_{B2} = \frac{(V_{B2})^2}{S_B^{1\Phi}} = 0.72\ \Omega$$

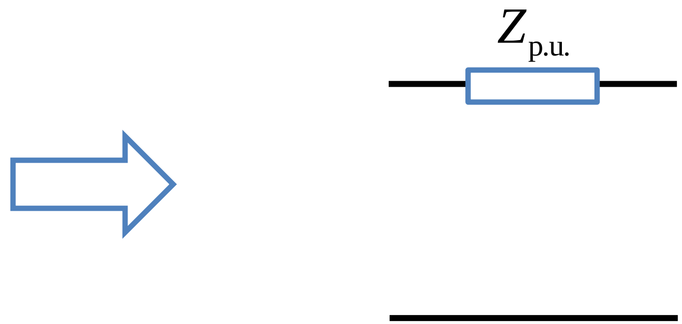
$$Z_{\mathrm{p.u.}} = \frac{Z_{eq2}}{Z_{B2}} = \frac{0.0525\angle 78.13°}{0.72}$$

$$= 0.0729\angle 78.13°\ \mathrm{p.u.}$$

Example 2: Per Unit Circuit

$S_B = 20\ \text{kVA}$





$V_{B1} = 480\ \text{V}$ $\quad$ $V_{B2} = 120\ \text{V}$

$Z_{B1} = 11.52\,\Omega$ $\quad$ $Z_{B2} = 0.72\,\Omega$

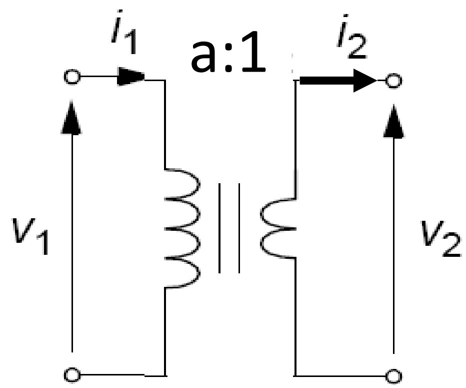
Per unit equivalent circuit

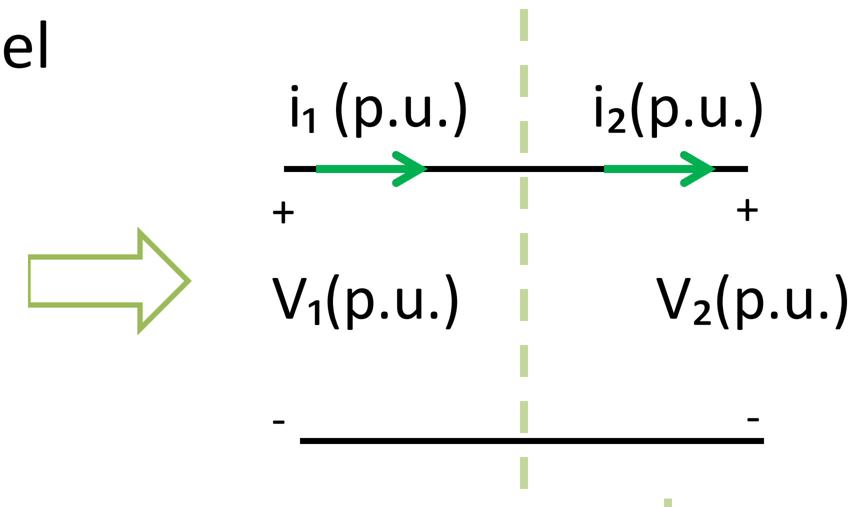
$$Z_{\text{p.u.}} = 0.0729\angle 78.13^\circ\ \text{p.u.}$$

There is no transformer in the per unit equivalent circuit.

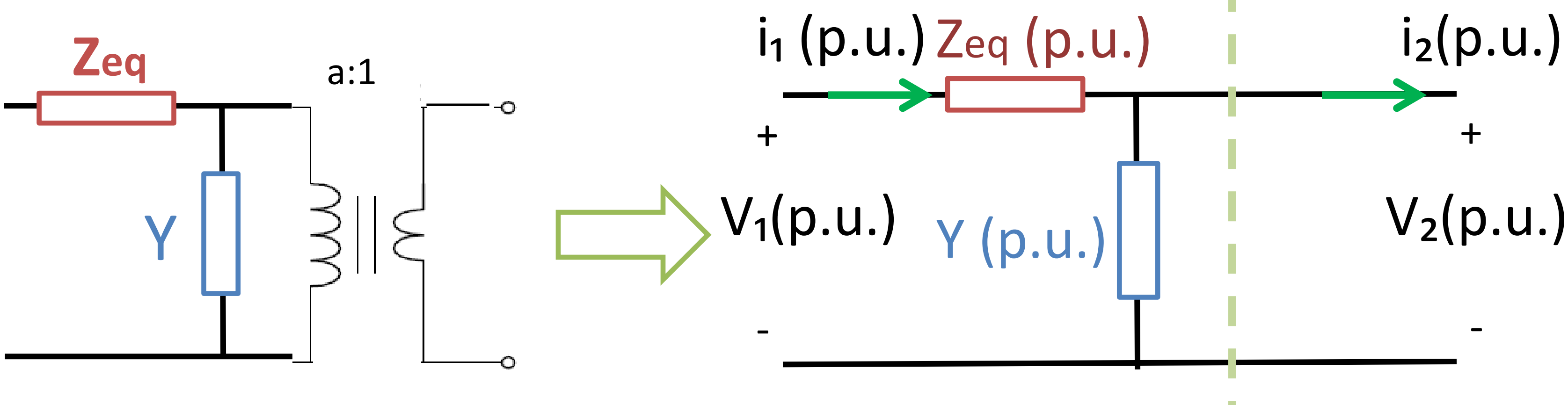
P.U. Equivalent Circuit of a Transformer

- Ideal transformer model





- Practical transformer model



Change of Base Value

- Manufacturers usually specify equipment impedances in per unit values together with voltage ratings (V) and apparent power rating (VA).
- The impedance base values can be found from the ratings of the equipment.
- Different equipment has different ratings.
- We may need to calculate per unit values on the new basis.

$$Z_{\text{actual}} = Z_{\text{p.u.}}^{\text{old}} Z_{\text{B}}^{\text{old}} = Z_{\text{p.u.}}^{\text{new}} Z_{\text{B}}^{\text{new}} \Rightarrow Z_{\text{p.u.}}^{\text{new}} = \frac{Z_{\text{p.u.}}^{\text{old}} Z_{\text{B}}^{\text{old}}}{Z_{\text{B}}^{\text{new}}}$$

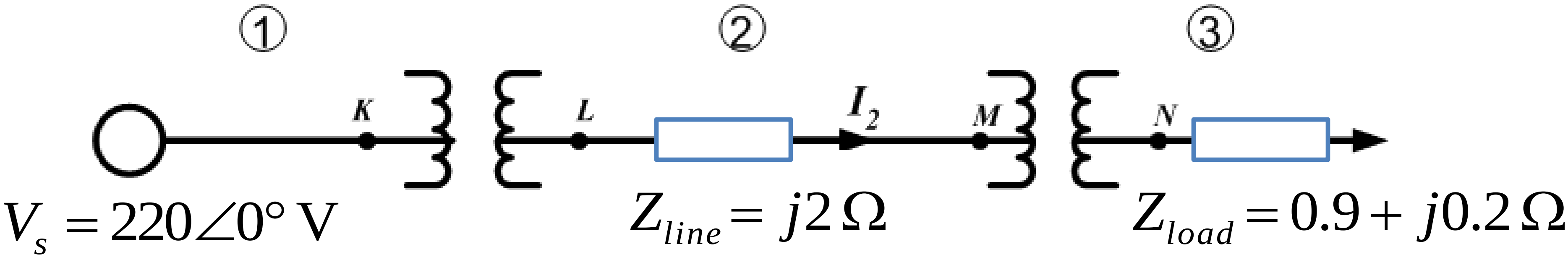
You should practice this in tutorial problems.

Steps of Per Unit Analysis

1. Choose $S_B^{1\Phi}$ for the system.
2. Select V_B for different zones (usually follows transformer voltage ratings).
3. Calculate Z_{B} for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Example 3: 1Φ, Per Unit Analysis

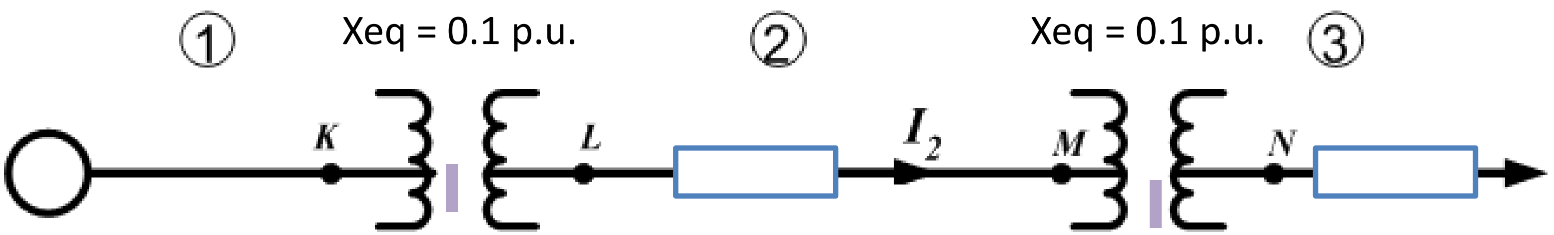
- Three zones of a single-phase circuit are shown below. Use base value of 30 kVA and 240 V in zone 1, draw per unit circuit and find per unit value of source voltage and all impedances.



T1: 30 kVA
240/480 V
Xeq = 0.1 p.u.

T2: 20 kVA
460/115 V
Xeq = 0.1 p.u.

Example 3: Base Values of Each Zone



$$S_B = 30\text{ kVA}$$

$$V_{B1} = 240\text{ V}$$

$$V_{s,p.u.} = \frac{220\angle 0^\circ}{240} = 0.9167\angle 0^\circ\text{ V}$$

$$V_{B2} = \left(\frac{480}{240}\right)240 = 480\text{ V}$$

$$Z_{B2} = \frac{(V_{B2})^2}{S_B} = 7.68\,\Omega$$

$$Z_{line,p.u.} = \frac{j2}{Z_{B2}} = j0.26042\text{ p.u.}$$

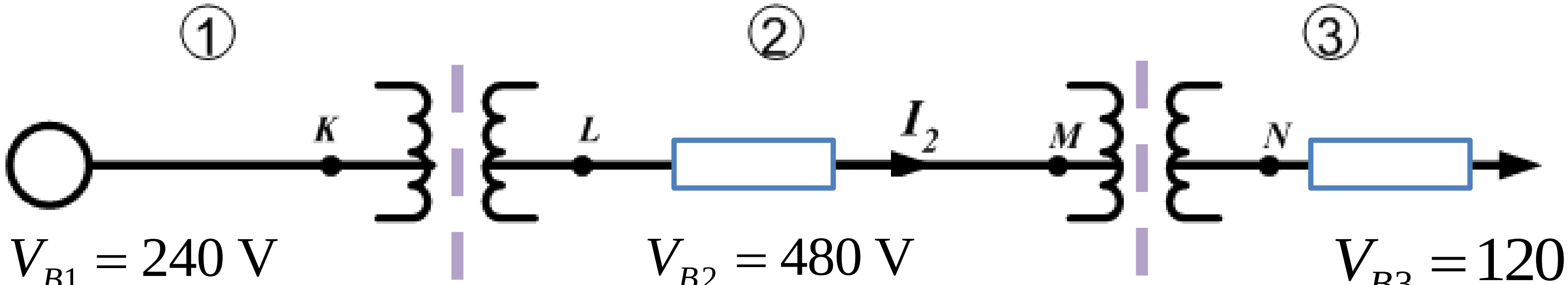
$$V_{B3} = \left(\frac{115}{460}\right)480 = 120\text{ V}$$

$$Z_{B3} = \frac{(V_{B3})^2}{S_B} = 0.48\,\Omega$$

$$Z_{load,p.u.} = \frac{0.9 + j0.2}{Z_{B2}} = 1.875 + j0.4167\text{ p.u.}$$

Example 5: P.U. Transformer Reactance

$$S_B = 30\text{ kVA}$$



$V_{B1} = 240\text{ V}$ | $V_{B2} = 480\text{ V}$ | $V_{B3} = 120\text{ V}$

T1: 30 kVA
240/480 V
Xeq = 0.1 p.u.

$Z_{B2} = 7.68\,\Omega$

T2: 20 kVA
460/115 V
Xeq = 0.1 p.u.

$Z_{B3} = 0.48\,\Omega$

For T1, S base and and V base are the same as of the circuit. In this case, we don't need to change the base. The per unit value of reactance is the same = 0.1 p.u.

$$X_{\text{T2,p.u.}}^{\text{new}} = \frac{X_{\text{p.u.}}^{\text{old}} X_{\text{B}}^{\text{old}}}{X_{\text{B}}^{\text{new}}} = \frac{0.1\left(\frac{115^2}{20000}\right)}{0.48} = 0.1378\text{ p.u.}$$

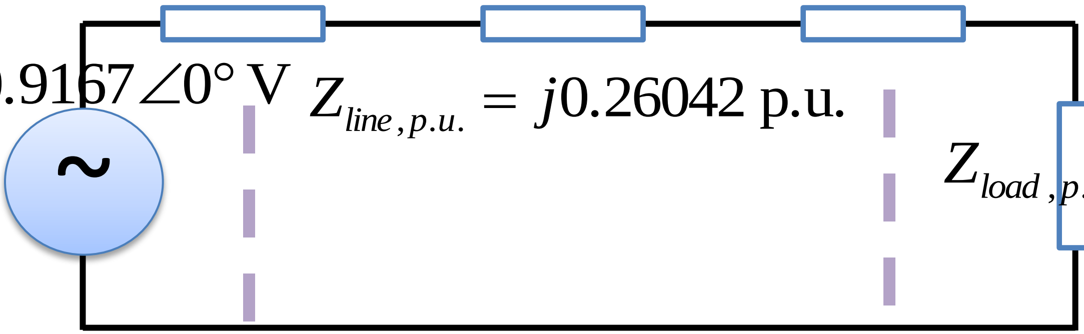
$$X_{\text{T2,p.u.}}^{\text{new}} = \frac{X_{\text{p.u.}}^{\text{old}} X_{\text{B}}^{\text{old}}}{X_{\text{B}}^{\text{new}}} = \frac{0.1\left(\frac{460^2}{20000}\right)}{7.68} = 0.1378\text{ p.u.}$$

Example 3: Per Unit Circuit

$$Z_{\text{T1,p.u.}} = j0.1\,\text{p.u.} \qquad Z^{\text{new}}_{\text{T2,p.u.}} = j0.1378\,\text{p.u.}$$

$$V_{s,p.u.} = 0.9167\angle 0^\circ\ \text{V} \qquad Z_{line,p.u.} = j0.26042\ \text{p.u.}$$

$$Z_{load,p.u.} = 1.875 + j0.4167\ \text{p.u.}$$



$$V_{B1} = 240\ \text{V} \qquad V_{B2} = 480\ \text{V} \qquad V_{B3} = 120\ \text{V}$$

$$I_{B1} = \frac{30000}{240} = 125\ \text{A} \qquad I_{B2} = \frac{30000}{480} = 62.5\ \text{A} \qquad I_{B2} = \frac{30000}{120} = 250\ \text{A}$$

$$I_{p.u.} = \frac{0.9167\angle 0^\circ}{j0.1 + j0.26042 + j0.1378 + 1.875 + j0.4167} = 0.4395\angle -26.01^\circ \text{V}$$

We can find current at any part of the circuit by simply multiplying the per unit value with the base value.

Advantage of Per Unit Analysis

- Simplify calculation by eliminating transformers.
- Helps to spot errors in data
  - p.u. is more uniform compare to actual impedance value of different sizes of equipment.
- Helps to detect abnormality in the system
  - Operator at control center can spot over/under voltage/current rating easily.

Inside CAISO

Source: http://www.youtube.com/watch?v=wDaoHNsX9lo

Next Lecture

- Three-phase transformer
  - Delta/Wye Connections
- Three-phase per unit analysis