EE2022 Electrical Energy Systems

Lecture 13: Transformer and Per Unit Analysis
02-03-2012

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Department of Electrical and Computer Engineering

Detailed Syllabus

20/01/2012	**Three-phase circuit analysis**: Introduction to three-phase circuit. Balanced three-phase systems.
20/01/2012	**Three-phase circuit analysis**: Delta-Wye connection, Relationship between phase and line quantities
27/01/2012	**Three-phase circuit analysis**: Per-phase analysis, Three-phase power calculation
27/01/2012	Guest Lecture "Energy & Environment, Smart Grid & Challenges Ahead" by Prof. J Nanda (IIT Delhi, IEEE Fellow)
30/01/2012	**Generator modeling**: Simple generator concept
03/02/2012	**Generator modeling**: Equivalent circuit of synchronous generators
03/02/2012	**Generator modeling**: Operating consideration of synchronous generators, i.e. Excitation voltage control, real power control, and loading capability
06/02/2012	**Generator modeling**: Principle of asynchronous generators
10/02/2012	**Transmission line modeling**: Overhead VS Underground cable
10/02/2012	**Transmission line modeling**: Four basic parameters of transmission line
13/02/2012	**Transmission line modeling**: Long transmission line model, Medium-length transmission line model, Short transmission line model
17/02/2012	**Transmission line modeling**: Operating consideration of transmission lines i.e. voltage regulation, line loadability, efficiency
17/02/2012	**Transformer and per unit analysis**: Principle of transformer, Single-phase transformer
27/02/2012	**Transformer and per unit analysis**: Single-phase per unit analysis
02/03/2012	**Transformer and per unit analysis**: Three-phase transformer, Three-phase per unit analysis
02/03/2012	**Review :** if time permits.

Learning Outcomes

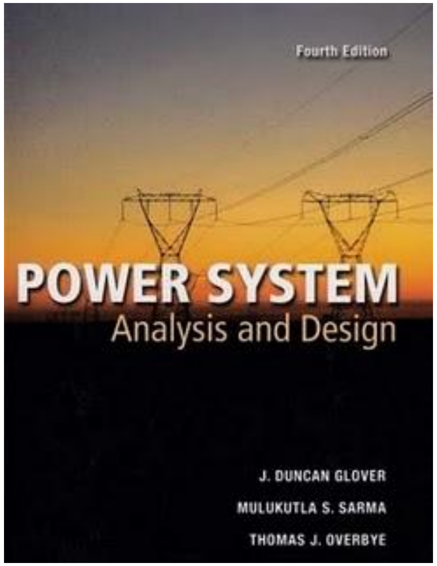
- Formulate equivalent circuits of various components in electrical energy systems
  – Equivalent circuit of transformer
- Analyze three-phase balanced circuits
  – Per-unit analysis for three-phase circuit

Outline

- Three Phase Transformer
  - Delta/Wye Connections
- Three-phase per unit analysis

References

- Glover, Sarma, and Overbye, “Power System Analysis and Design”.
  - Chapter 3

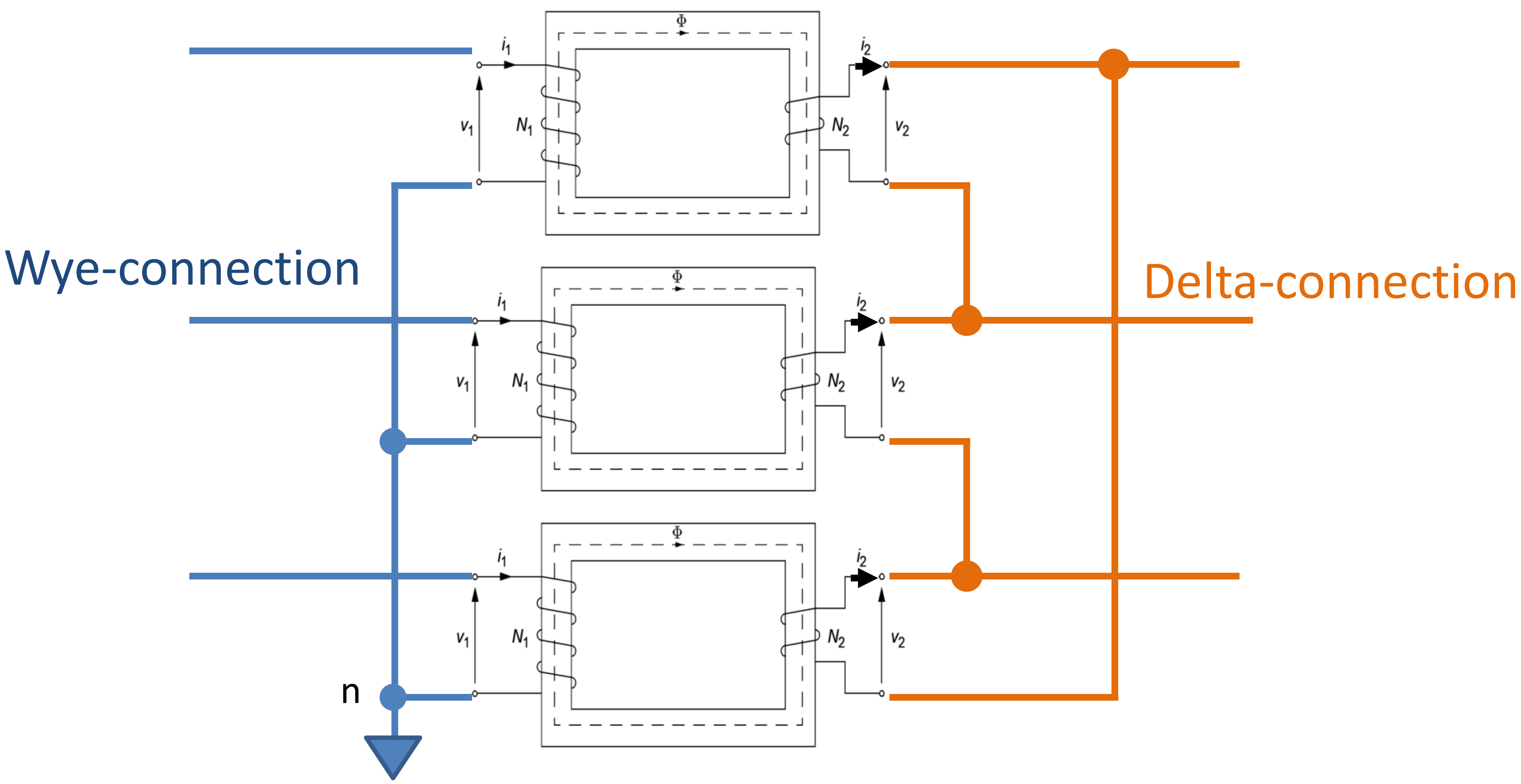


Three-phase transformer connections

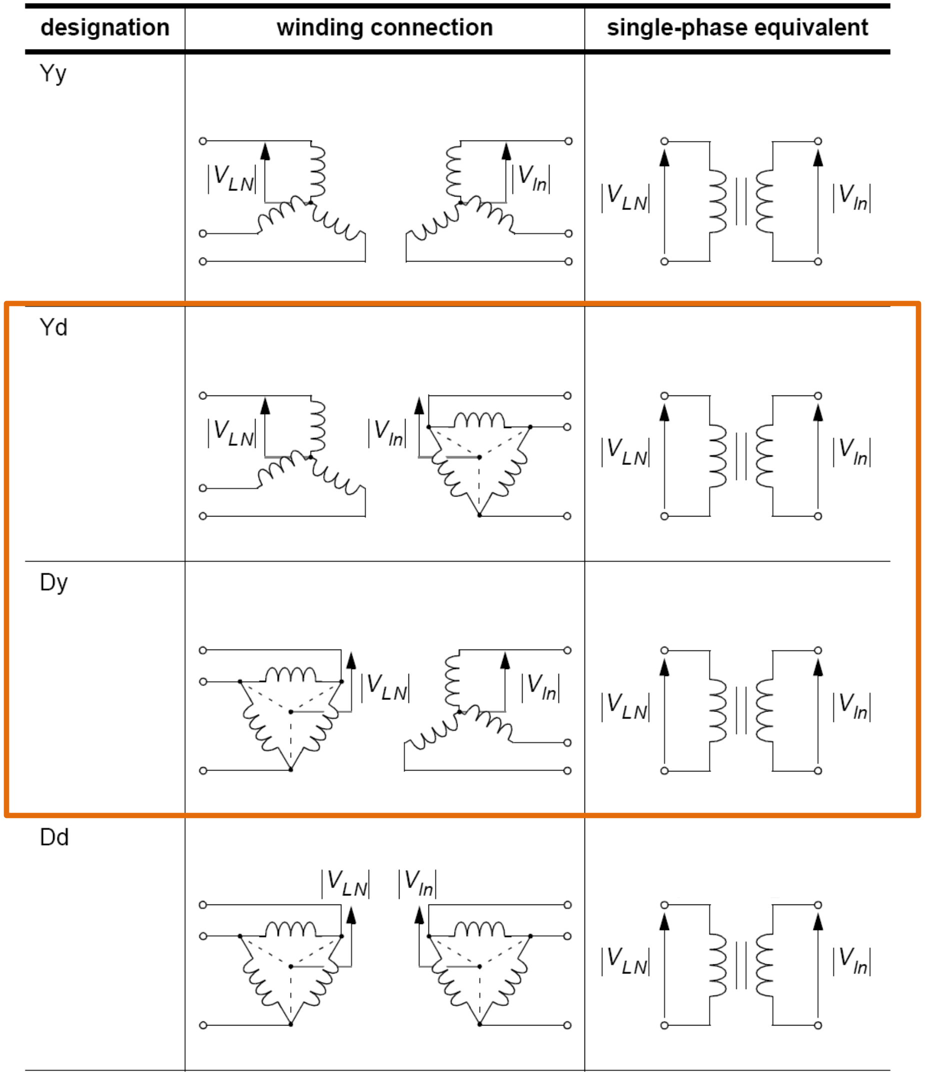
Phase shift

THREE PHASE TRANSFORMER

Three Single-Phase Transformers

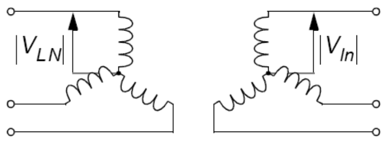
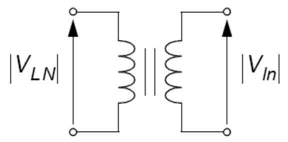
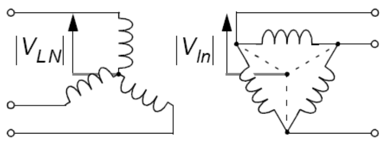
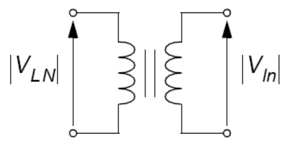
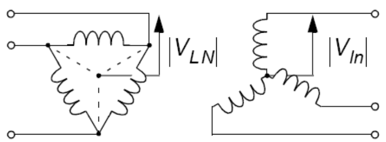
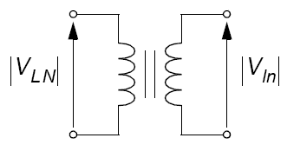
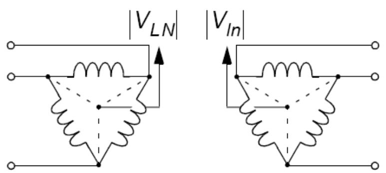
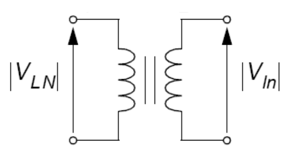


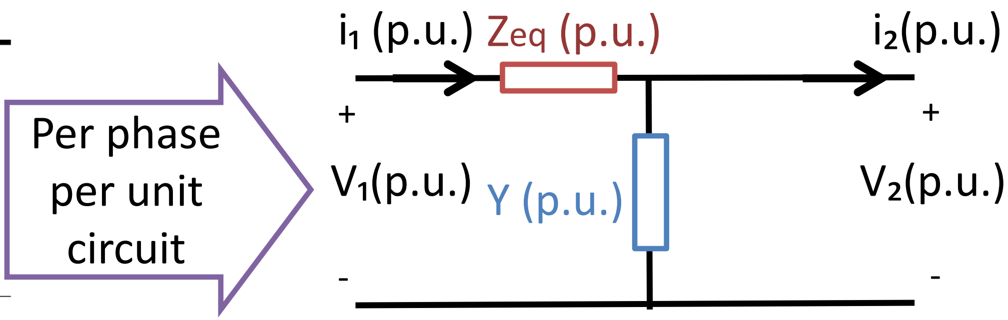
3Φ Transformer Connection



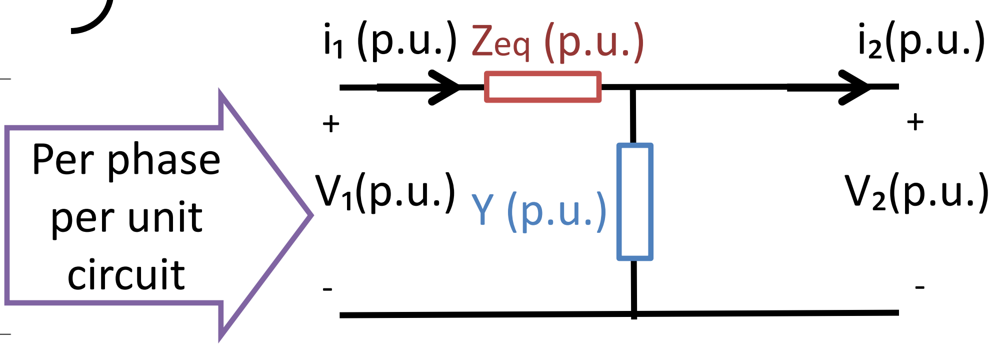
- The voltage rating of a three phase transformer is the ratio between **line-to-line** voltage at the primary side and **line-to-line** voltage at the secondary side.
- The single-phase equivalent shows line-to-neutral voltage.
- For Y-Y and Δ-Δ transformers, voltage and current in both primary and secondary are in phase. The ratio of the voltage and current follows the turn ratio of the transformer.
- The same does not apply to Y-Δ and Δ-Y connections.

3Φ Transformer Connection

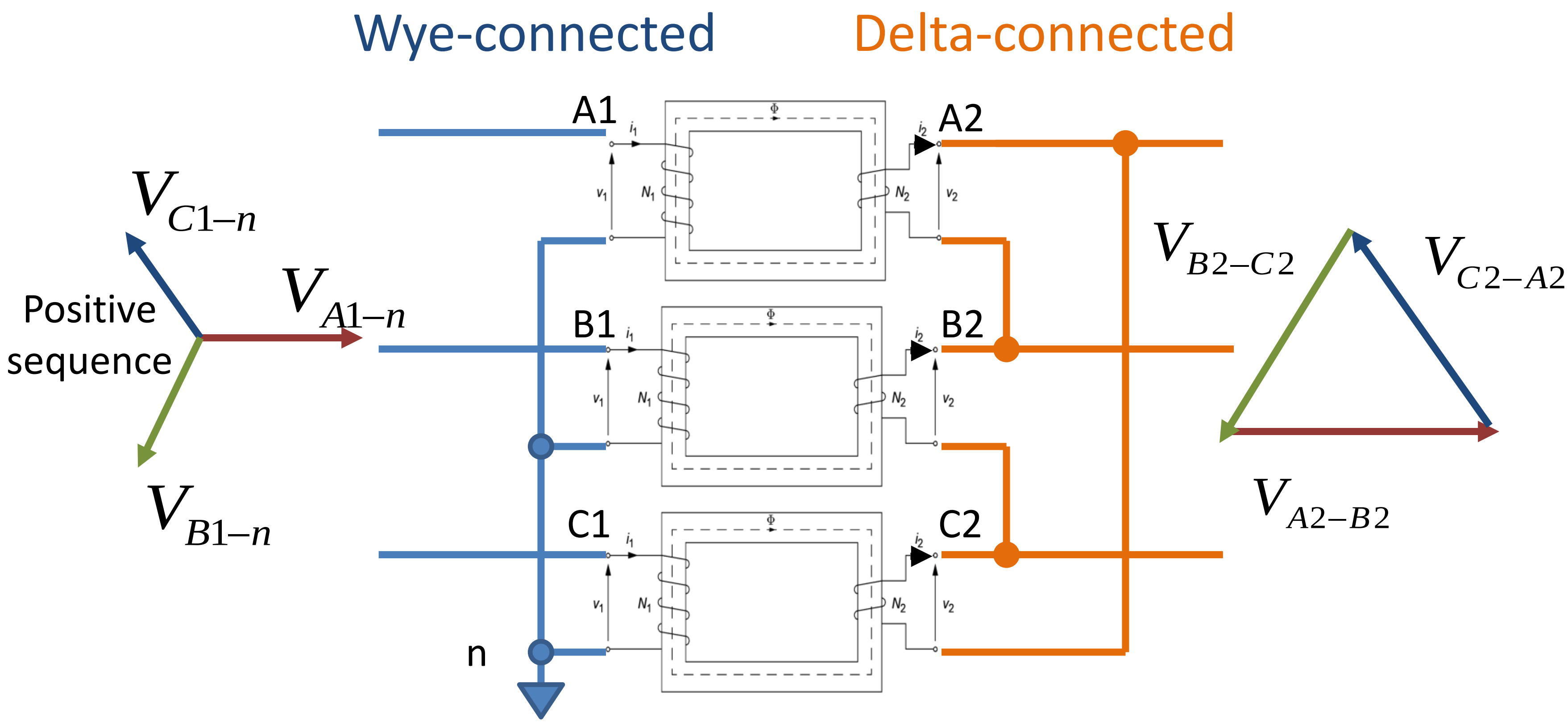
designation	winding connection	single-phase equivalent
Yy		
Yd		
Dy		
Dd		



Wye-Delta and Delta-Wye transformer connections introduce some phase shift to the voltage.



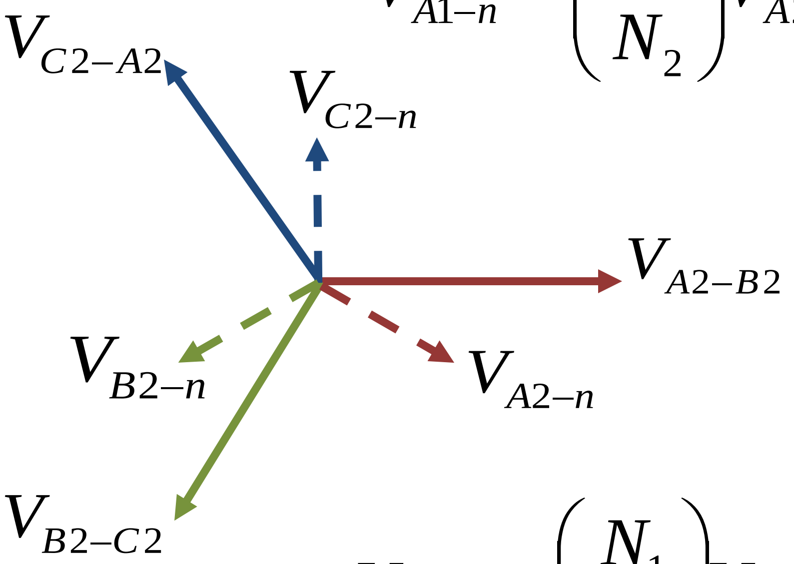
Wye-Delta 3Φ Transformer



Line-to-neutral voltage $V_{A1-n} = \left(\frac{N_1}{N_2}\right) V_{A2-B2}$ *Line-to-line voltage!!*

Wye-Delta 3Φ Transformer

$$V_{A1-n} = \left(\frac{N_1}{N_2}\right) V_{A2-B2}$$



For a positive sequence,

$$V_{A2-B2} = V_{A2-n} - V_{B2-n}$$

$$= V_{A2-n} - V_{A2-n}\angle -120°$$

$$= \sqrt{3} V_{A2-n}\angle 30°$$

$$V_{A1-n} = \left(\frac{N_1}{N_2}\right) V_{A2-B2} = \left(\frac{N_1}{N_2}\right) \sqrt{3} V_{A2-n}\angle 30°$$

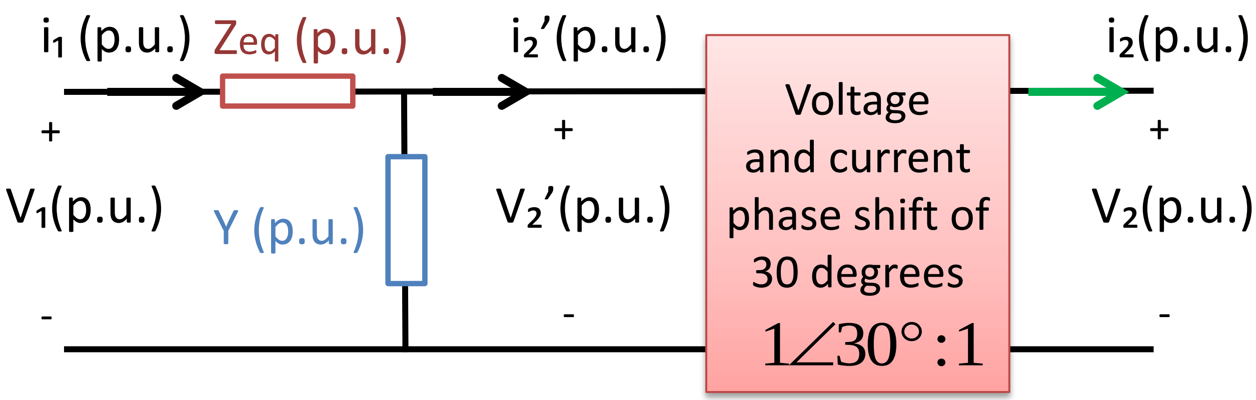
This value becomes the given turn ratio of the transformer.

The line-to-neutral voltage at the primary side leads the line-to neutral voltage at the secondary side by 30 degrees. The same is true for the current.

Y-Δ 3Φ Transformer: Per Phase Model

For a positive sequence voltage source,

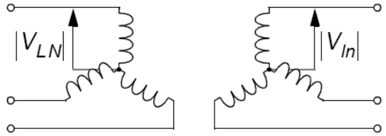
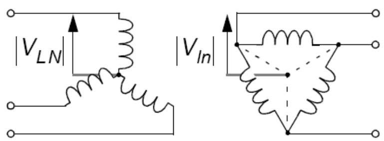
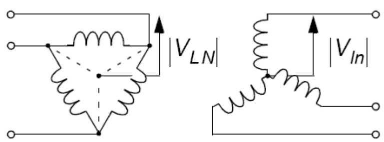
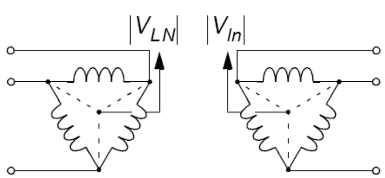
$$V_{A1-n} = \left(\frac{N_1}{N_2}\right)\sqrt{3}V_{A2-n}\angle 30^\circ \quad \Rightarrow \quad \angle V_{A1-n} : \angle V_{A2-n} = 1\angle 30^\circ : 1$$

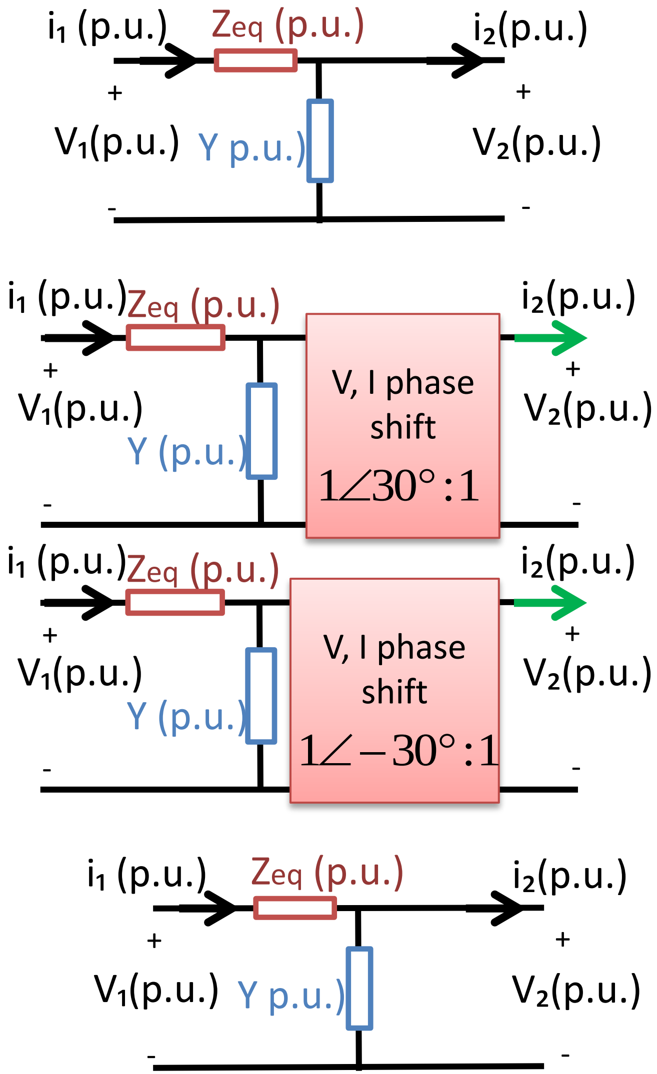


$$i_2 = i_2'\angle -30^\circ$$

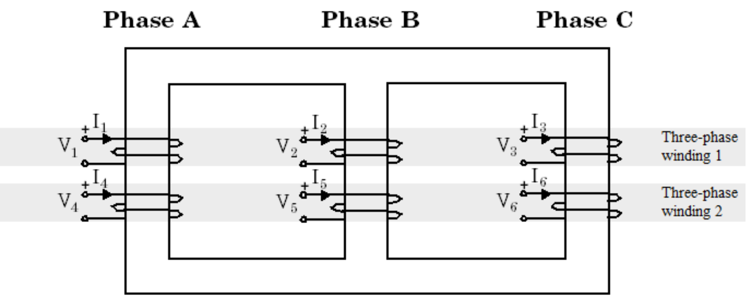
$$V_2 = V_2'\angle -30^\circ$$

3Φ Transformer Per Unit Model

designation	winding connection
Yy	
Yd	
Dy	
Dd	

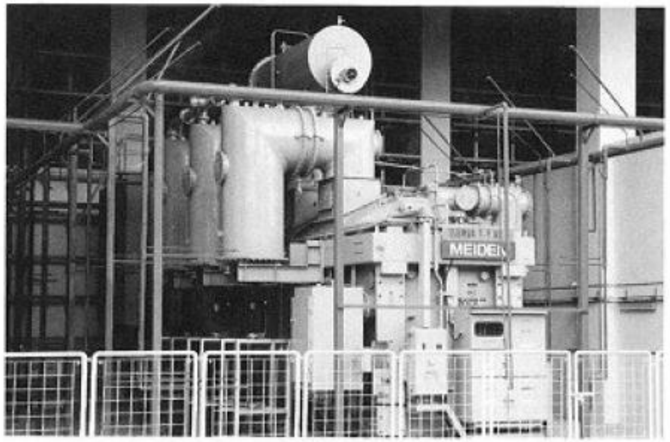


Three-Phase Transformer



Source: www.mathworks.com

230 kV, 150 MVA oil-immersed forced-air cooled transformer at Kampong Java substation, PowerGrid, Singapore

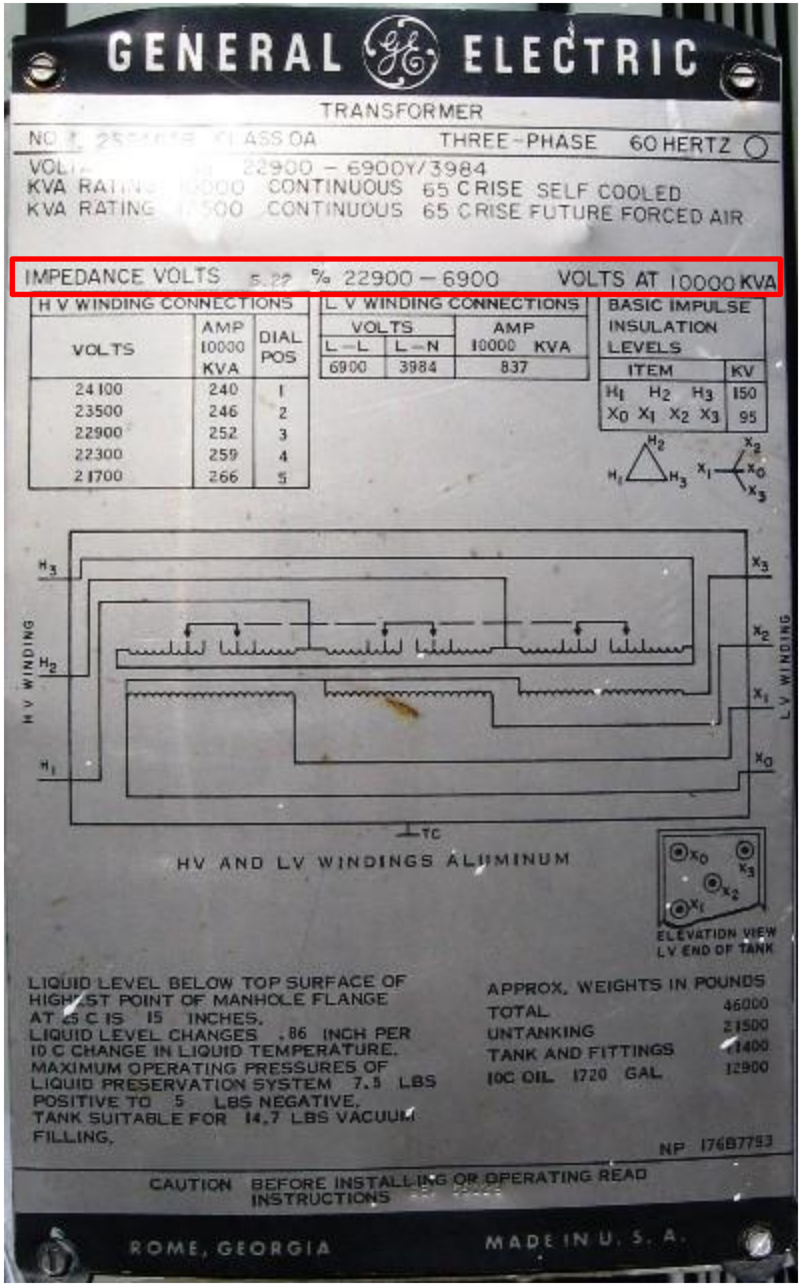


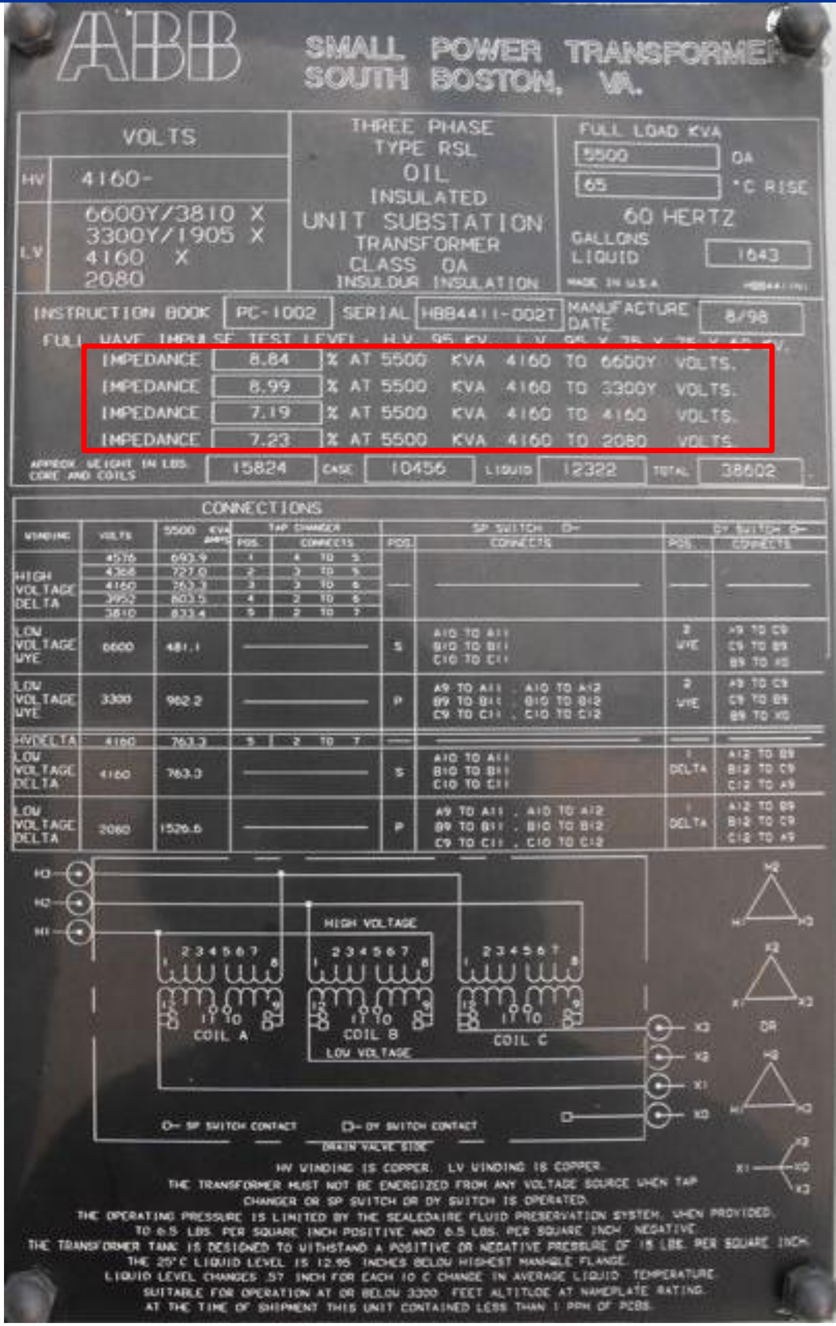
Source: http://www.meidensg.com.sg

Source: http://www.youtube.com/watch?v=9Y958Vc5ohI&feature=BFa&list=PLC7AEFDAFD9223780&lf=rellist

5.5 MVA ABB and 10 MVA GE Substation Transformer Nameplate

Source: http://www.tucsontransformer.com





Per Unit Quantity: 3Φ

Base Value: 3Φ

Steps of Calculation: 3Φ

THREE PHASE PER UNIT ANALYSIS

Recall: Per Unit Quantity

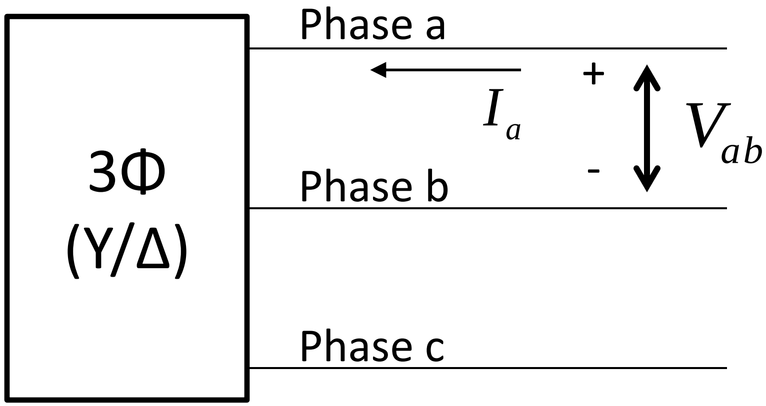
- The per unit quantity of voltage, current, power and impedance is found from dividing the actual quantity by a base value of that quantity.

$$\text{per}-\text{unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

- Per unit value is denoted by ‘p.u.’.
- The base value quantity typically follows transformer rating.
- The per unit values are the same irrespective of the sides of the transformer.

Per Unit Analysis in 3Φ Circuit

- Recall that in three phase circuit:
  - Voltage is given as line-to-line voltage.
  - Current is given as line current.
  - Apparent power is given as three-phase power.



$$\left|S_{3\Phi}\right| = \sqrt{3}\left|V_{\text{Line-To-Line}}\right|\left|I_{\text{Line}}\right|$$

Base Value Selection: 3Φ Case

- Base values are **real numbers**, denote by subscript '*B*'.
- Voltage base values follow transformer voltage ratings.
  - The voltage ratios are given as line-to-line voltage at both ends. This means that it already incorporates effects of the Y or Δ configurations.

$$V_B^{line-to-neutral} = \frac{V_B^{line-to-line}}{\sqrt{3}}$$

- Only **single** base complex power $S_B^{3\Phi}$ in the system.
- The base value of power is used to normalize the quantity. Thus, the base values of real power, reactive power, and complex power are all the same real number.

$$P_B^{3\Phi} = Q_B^{3\Phi} = S_B^{3\Phi}$$

Base Value Selection: 3Φ Case

- Current base values are calculated from the base power and base voltage.

$$I_B = \frac{S_B^{3\Phi}}{\sqrt{3}V_B^{line-to-line}} = \frac{S_B^{1\Phi}}{V_B^{line-to-neutral}}$$

- Impedance base values (same value for impedance, resistance, or reactance) are calculated from voltage and current.

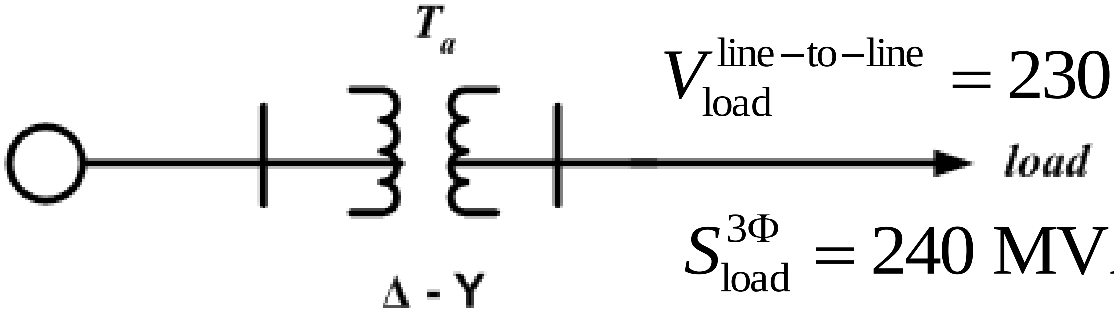
$$Z_B = \frac{V_B^{line-to-neutral}}{I_B} = \frac{\left(V_B^{line-to-neutral}\right)^2}{S_B^{1\Phi}} = \frac{\left(V_B^{line-to-line}\right)^2}{S_B^{3\Phi}}$$

Steps of Calculation: 3Φ Case

1. Choose $S_B^{3\Phi}$ for the system.
2. Select $V_B^{line-to-neutral}$ or $V_B^{line-to-line}$ for different zones.
3. Calculate Z_B for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Example 1: Δ-Y 3Φ Transformer

- Three-phase generator rated 300 MVA,23 kV supplying a system load of 240 MVA, 0.9 power factor lagging at 230 kV through a 330 MVA 23Δ/ 230Y kV **step-up** transformer of leakage reactance 11%.
- Use **base values at the load of 100 MVA and 230 kV**, find the current supplied by the generator. Use *load voltage for an angle reference*.



$$V_{\text{load}}^{\text{line-to-line}} = 230\text{ kV}$$

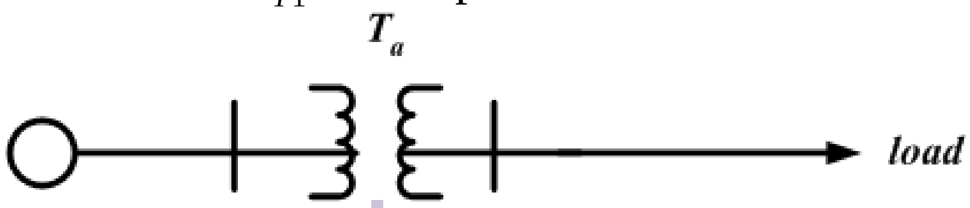
$$S_{\text{load}}^{3\Phi} = 240\text{ MVA}, 0.9\text{ pf lagging}$$

Example 1: Base Values

$$S_B^{3\Phi} = 100 \text{ MVA}$$

330 MVA
23Δ/ 230Y kV

$$X_{T1} = 0.11 \text{ p.u.}$$



$$V_{B1}^{\text{line-to-line}} = 23 \text{ kV}$$

$$V_{B1}^{\text{line-to-neutral}} = \frac{23}{\sqrt{3}} \text{ kV}$$

$$Z_{B1} = \frac{\left(V_{B1}^{\text{line-to-line}}\right)^2}{S_B^{3\Phi}} = \frac{\left(23 \times 10^3\right)^2}{100 \times 10^6} = 5.29\ \Omega$$

$$I_{B1} = \frac{S_B^{3\Phi}}{\sqrt{3} V_{B1}^{line-to-line}} = \frac{100 \times 10^6}{\sqrt{3} \times 23 \times 10^3} = 2510.2 \text{ A}$$

$$V_{B2}^{\text{line-to-line}} = 230 \text{ kV}$$

$$V_{B2}^{\text{line-to-neutral}} = \frac{230}{\sqrt{3}} \text{ kV}$$

$$Z_{B2} = \frac{\left(V_{B2}^{\text{line-to-line}}\right)^2}{S_B^{3\Phi}} = \frac{\left(230 \times 10^3\right)^2}{100 \times 10^6} = 529\ \Omega$$

$$I_{B2} = \frac{S_B^{3\Phi}}{\sqrt{3} V_{B2}^{line-to-line}} = \frac{100 \times 10^6}{\sqrt{3} \times 230 \times 10^3} = 251.02 \text{ A}$$

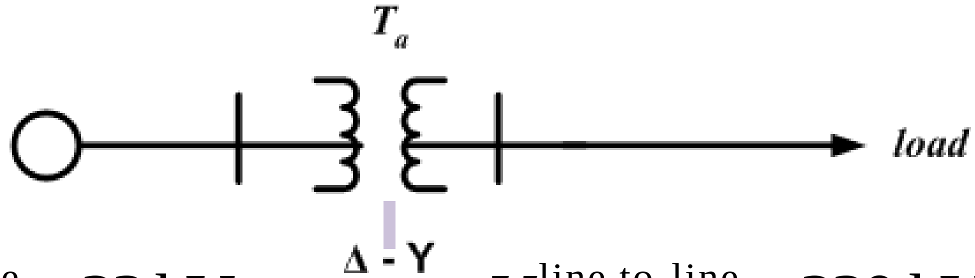
Example 1: Transformer Reactance

$$S_B^{3\Phi} = 100\text{ MVA}$$

330 MVA

23Δ/ 230Y kV

$X_{T1} = 0.11\text{ p.u.}$



$$V_{B1}^{\text{line-to-line}} = 23\text{ kV} \qquad V_{B2}^{\text{line-to-line}} = 230\text{ kV}$$

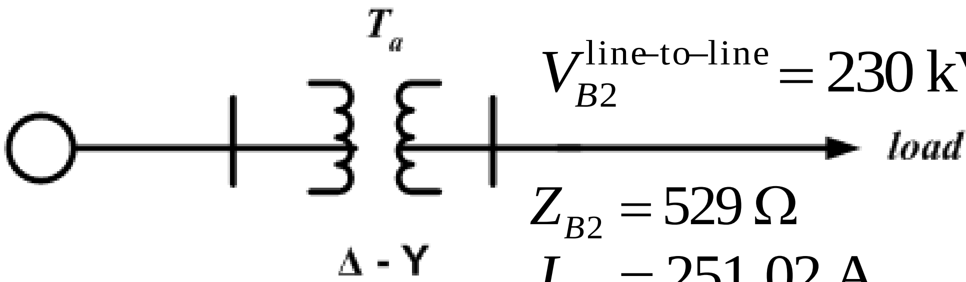
$$Z_{B1} = 5.29\,\Omega \quad I_{B1} = 2510.2\text{ A} \qquad Z_{B2} = 529\,\Omega \quad I_{B2} = 251.02\text{ A}$$

$$X_{\text{T,p.u.}}^{\text{new}} = \frac{X_{\text{p.u.}}^{\text{old}} X_{\text{B}}^{\text{old}}}{X_{\text{B}}^{\text{new}}} = \frac{0.11\left(\frac{(230\times10^3)^2}{330\times10^6}\right)}{529} = 0.03333\text{ p.u.}$$

$$X_{\text{T,p.u.}}^{\text{new}} = \frac{X_{\text{p.u.}}^{\text{old}} X_{\text{B}}^{\text{old}}}{X_{\text{B}}^{\text{new}}} = \frac{0.11\left(\frac{(23\times10^3)^2}{330\times10^6}\right)}{5.29} = 0.03333\text{ p.u.}$$

Example 1: Load Voltage and Current

$$S_B^{3\Phi} = 100 \text{ MVA}$$



$$V_{B2}^{\text{line-to-line}} = 230 \text{ kV}$$

$$Z_{B2} = 529\ \Omega$$

$$I_{B2} = 251.02 \text{ A}$$

At the load, use load voltage as reference.

$$V_{\text{load}}^{\text{line-to-line}} = 230\angle 0^\circ \text{ kV}$$

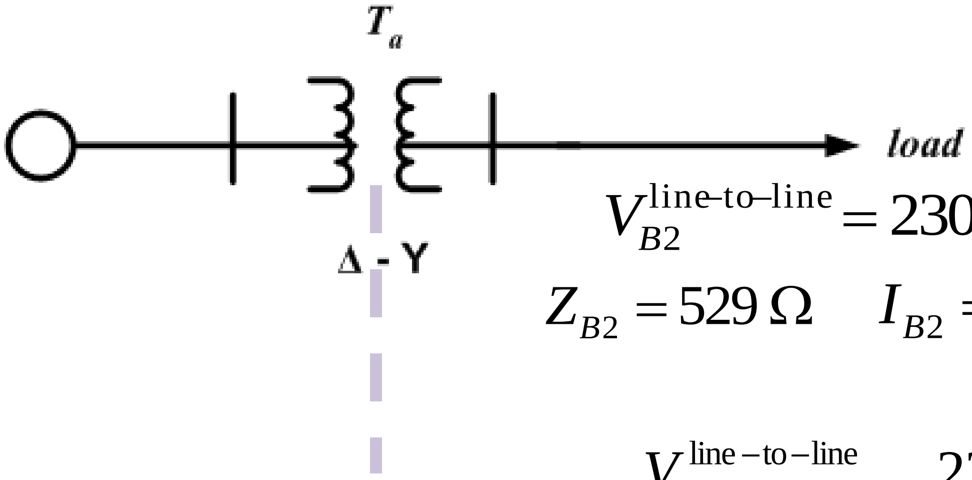
Load current magnitude is found from the three-phase apparent power and line-to-line voltage.

$$|I_{\text{load}}| = \frac{\left|S_{\text{load}}^{3\Phi}\right|}{\sqrt{3}\left|V_{\text{load}}^{\text{line-to-line}}\right|} = \frac{240\times 10^6}{\sqrt{3}\times 230\times 10^3} = 602.45 \text{ A}$$

Load current angle can be found from power factor. Note that the reference angle of load voltage is zero degree.

$$\angle I_{\text{load}} = \angle -\cos^{-1}(0.9)^\circ = -25.842^\circ$$

Example 1: Per Unit Values



$$V_{B2}^{\text{line-to-line}} = 230\ \text{kV}$$

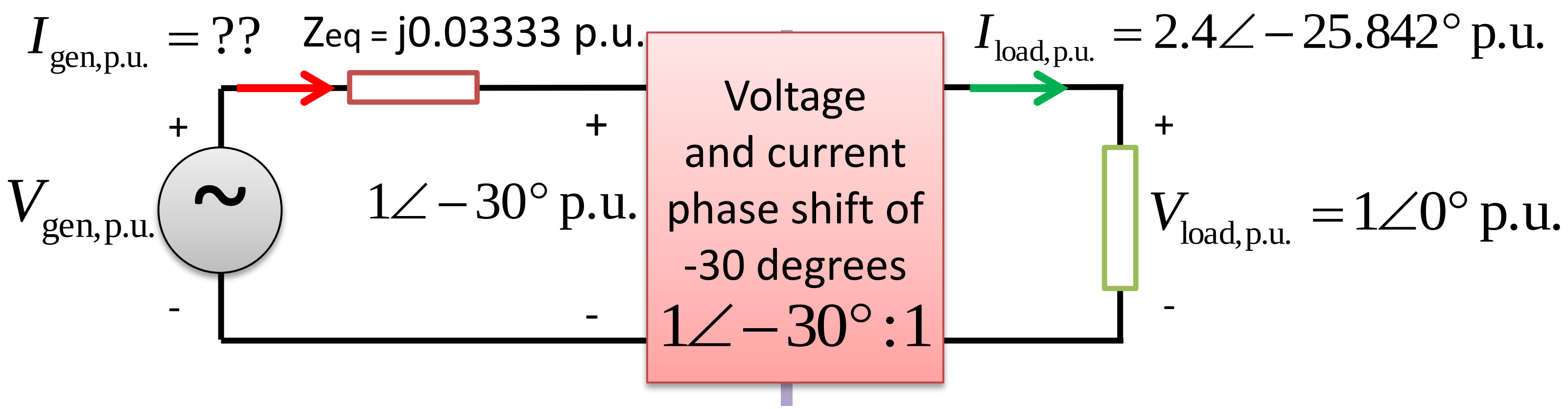
$$Z_{B2} = 529\ \Omega \qquad I_{B2} = 251.02\ \text{A}$$

$$V_{\text{load,p.u.}} = \frac{V_{\text{load}}^{\text{line-to-line}}}{V_{B2}^{\text{line-to-line}}} = \frac{230\angle 0^\circ\ \text{kV}}{230\ \text{kV}} = 1\angle 0^\circ\ \text{p.u.}$$

$$I_{\text{load,p.u.}} = \frac{I_{\text{load}}}{I_{B2}} = \frac{602.45\angle -25.842^\circ\ \text{A}}{251.02\ \text{A}} = 2.4\angle -25.842^\circ\ \text{p.u.}$$

Example 1: Per Unit Circuit

$I_{B1} = 2510.2\text{ A} \qquad I_{B2} = 251.02\text{ A}$



$$\frac{I_{\text{gen,p.u.}}}{I_{\text{load,p.u.}}} = \frac{1\angle -30^\circ}{1}$$

$$I_{\text{gen,p.u.}} = I_{\text{load,p.u.}}\angle -30^\circ = 2.4\angle -55.842^\circ$$

$$I_{\text{gen}} = I_{\text{gen,p.u.}} \times I_{B1} = 2.4\angle -55.842^\circ \times 2510.2 = 6024.5\angle -55.842^\circ\text{ A}$$

Can you find the line-to-line voltage supplied by the generator?

Ans: $V_{\text{gen}}^{\text{line-to-line}} = 23.86\angle -26.02\text{ kV}$

Summary

- Per unit system helps to eliminate transformer in the circuit analysis.
- Per unit value is found by normalize the actual value by base value.
- We can divide circuits into zones according to transformer voltage ratings.
- Choose only single base power. Voltage, current, and impedance base value is calculated for each zone.
- Actual value is found from multiplying the per unit value to its corresponding base value in its zone.
- Three-phase transformer Y-Δ and Δ-Y connections introduces phase shift of 30 and -30 degrees correspondingly.

Subsequent Lectures

1. **Introduction –** Electrical energy systems
2. **AC systems basics** - Active, Reactive and Apparent Power; Concept of Power factor, lagging, leading and unity power factor operation;
3. **Single- and three-phase power system**; Star and Delta connection; Relationship between phase and line quantities; per-unit representation.
4. **Generation, Transmission and Distribution Network** - Power system structure; Generation, Transmission and Distribution; Transmission line modeling; Complex power transmission; Transformer, Principle of operation of synchronous and asynchronous machines; Applications.
5. **Renewable Energy Sources** - Sustainable and clean energy sources; Solar Photovoltaic, Wind, Hydro, Fuel-cell
6. **Distributed Generation** - Concept of distributed energy generation and utility interfacing; Smart Grid
7. **Energy storage** - Concept of energy storage; Batteries, Super-capacitors, Fly-wheels
8. **Singapore Electricity Network** - Singapore power system network, Electricity system unbundling; **Electric energy market** structure and operation; Cost of electricity

1^{st} Mid-term Test

- March 5^{th} 5:15pm to 6:00pm @LT 4
- Materials (Lecture 1-5, Tutorials 1-3):
  - AC circuit analysis
  - Apparent power, power factor correction
  - Three-phase voltage and current
  - Three-phase complex power
- Format:
  - 4 questions, closed book.
  - 45 minutes.