

EE2022 Electrical Energy Systems

Lecture 13: Transformer and Per Unit Analysis 02-03-2012

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Detailed Syllabus

20/01/2012	Three-phase circuit analysis: Introduction to three-phase circuit. Balanced three-phase systems.
20/01/2012	Three-phase circuit analysis: Delta-Wye connection, Relationship between phase and line quantities
27/01/2012	Three-phase circuit analysis: Per-phase analysis, Three-phase power calculation
27/01/2012	Guest Lecture “Energy & Environment, Smart Grid & Challenges Ahead” by Prof. J Nanda (IIT Delhi, IEEE Fellow)
30/01/2012	Generator modeling: Simple generator concept
03/02/2012	Generator modeling: Equivalent circuit of synchronous generators
03/02/2012	Generator modeling: Operating consideration of synchronous generators, i.e. Excitation voltage control, real power control, and loading capability
06/02/2012	Generator modeling: Principle of asynchronous generators
10/02/2012	Transmission line modeling: Overhead VS Underground cable
10/02/2012	Transmission line modeling: Four basic parameters of transmission line
13/02/2012	Transmission line modeling: Long transmission line model, Medium-length transmission line model, Short transmission line model
17/02/2012	Transmission line modeling: Operating consideration of transmission lines i.e. voltage regulation, line loadability, efficiency
17/02/2012	Transformer and per unit analysis: Principle of transformer, Single-phase transformer
27/02/2012	Transformer and per unit analysis: Single-phase per unit analysis
02/03/2012	Transformer and per unit analysis: Three-phase transformer, Three-phase per unit analysis
02/03/2012	Review : if time permits.

Learning Outcomes

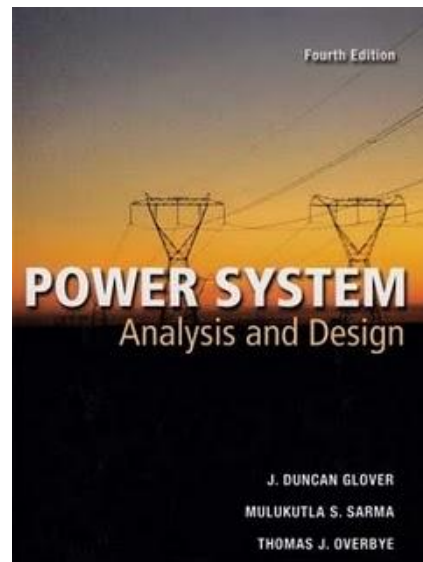
- Formulate equivalent circuits of various components in electrical energy systems
 - Equivalent circuit of transformer
- Analyze three-phase balanced circuits
 - Per-unit analysis for three-phase circuit

Outline

- Three Phase Transformer
 - Delta/Wye Connections
- Three-phase per unit analysis

References

- Glover, Sarma, and Overbye, “Power System Analysis and Design”.
 - Chapter 3

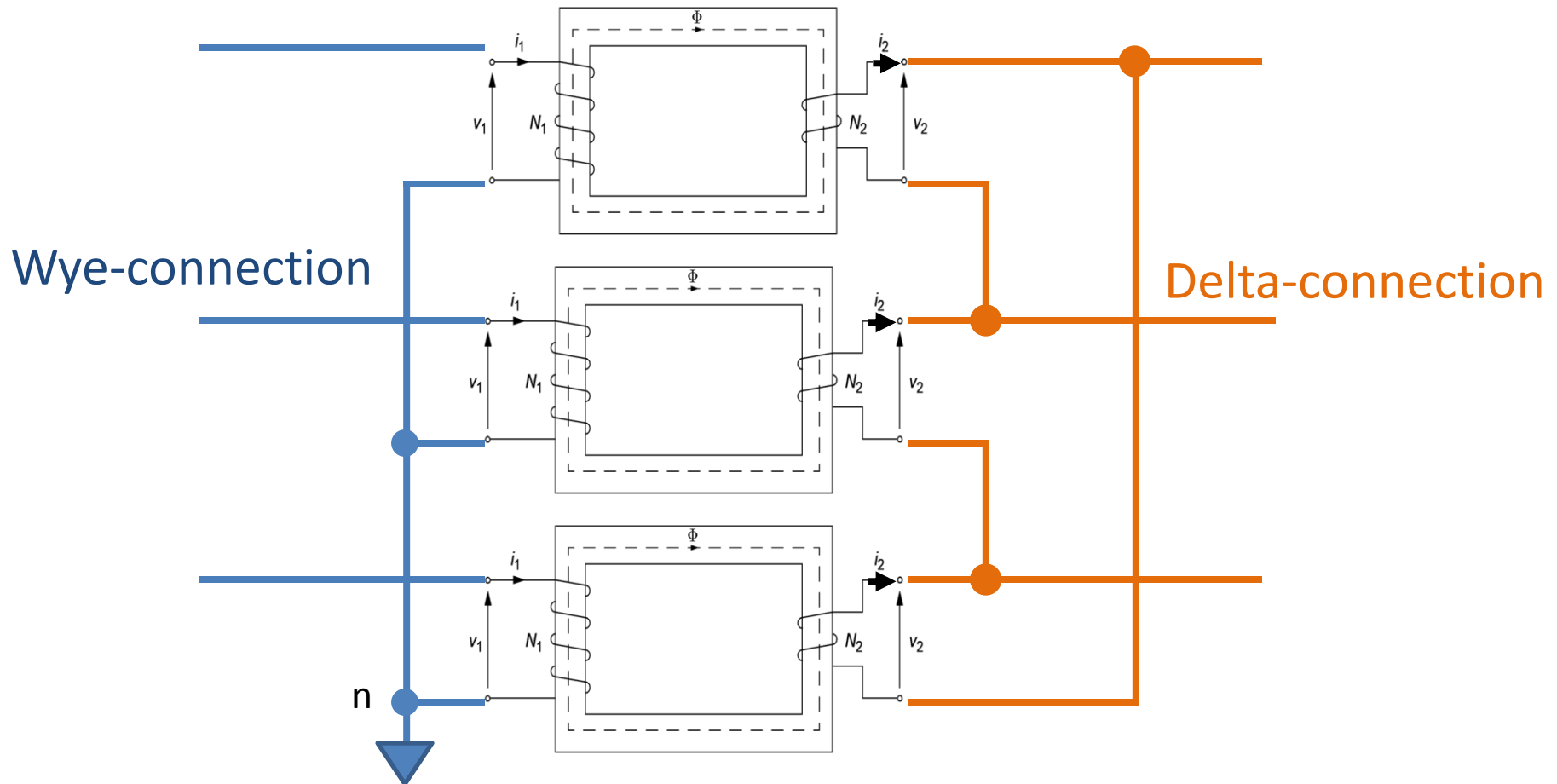


Three-phase transformer connections

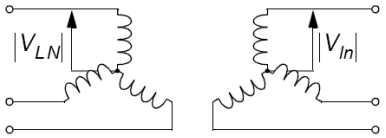
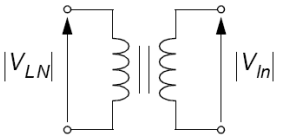
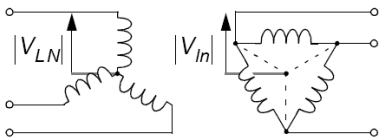
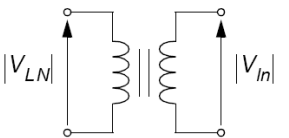
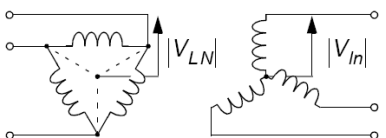
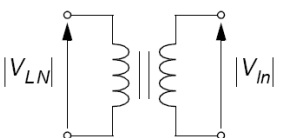
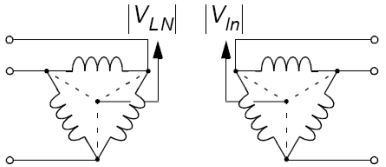
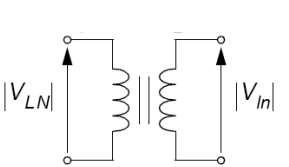
Phase shift

THREE PHASE TRANSFORMER

Three Single-Phase Transformers

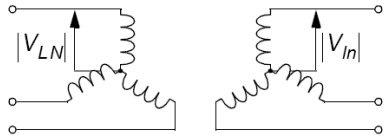
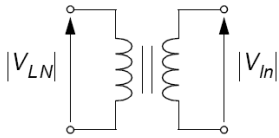
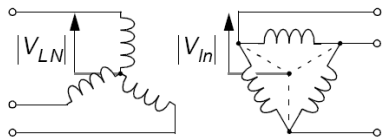
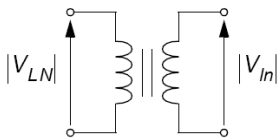
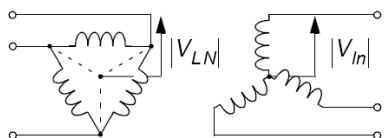
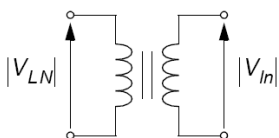
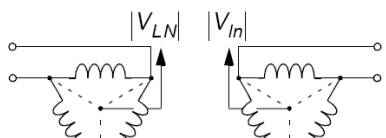
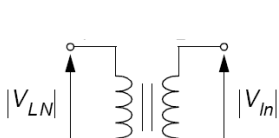


3 Φ Transformer Connection

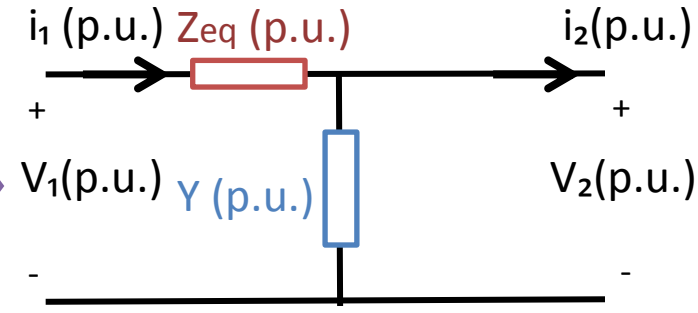
designation	winding connection	single-phase equivalent
Yy		
Yd		
Dy		
Dd		

- The voltage rating of a three phase transformer is the ratio between **line-to-line** voltage at the primary side and **line-to-line** voltage at the secondary side.
- The single-phase equivalent shows line-to-neutral voltage.
- For Y-Y and Δ - Δ transformers, voltage and current in both primary and secondary are in phase. The ratio of the voltage and current follows the turn ratio of the transformer.
- The same does not apply to Y- Δ and Δ -Y connections.

3Φ Transformer Connection

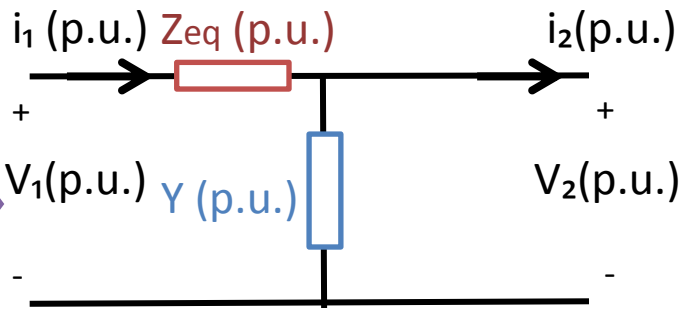
designation	winding connection	single-phase equivalent
Yy		
Yd		
Dy		
Dd		

Per phase per unit circuit



} *Wye-Delta and Delta-Wye transformer connections introduce some phase shift to the voltage.*

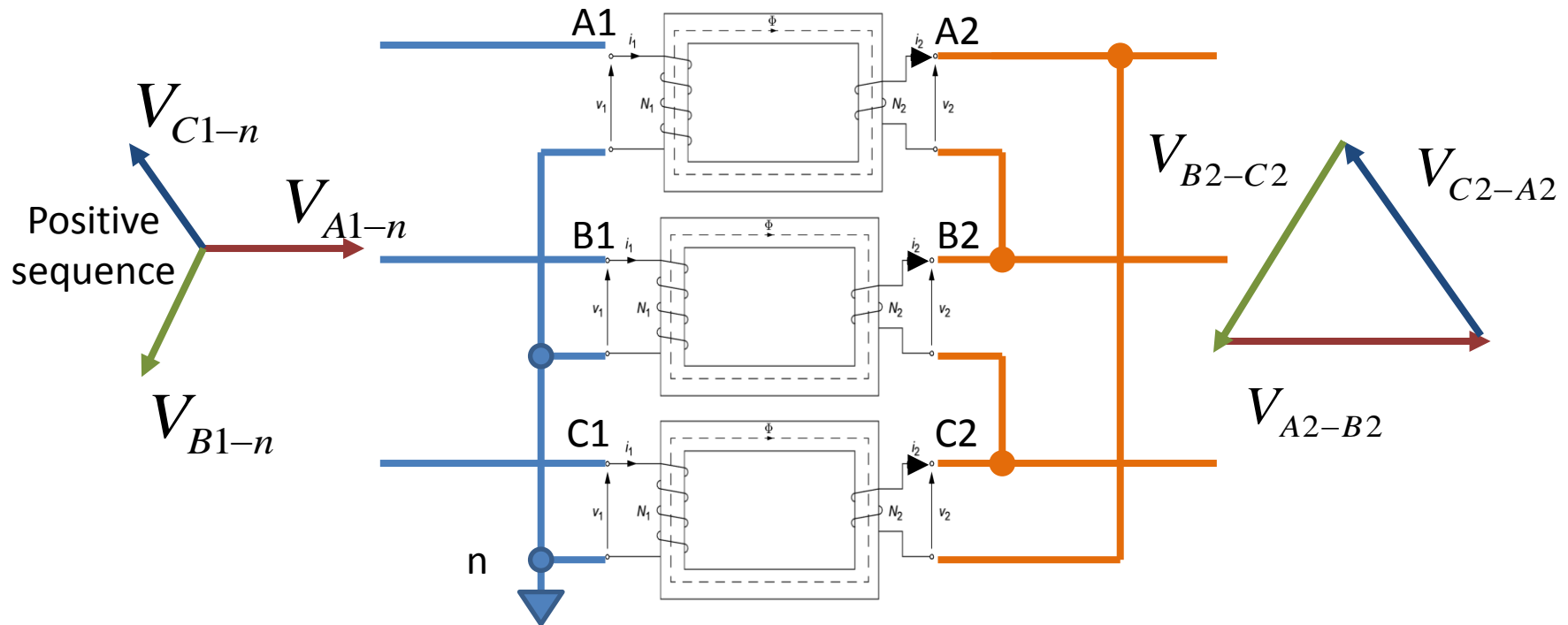
Per phase per unit circuit



Wye-Delta 3 Φ Transformer

Wye-connected

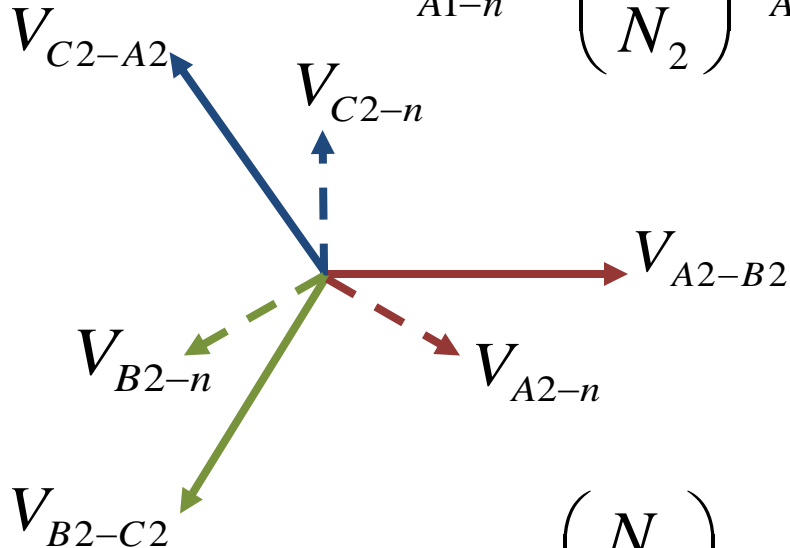
Delta-connected



Line-to-neutral voltage $V_{A1-n} = \left(\frac{N_1}{N_2} \right) V_{A2-B2}$ *Line-to-line voltage!!*

Wye-Delta 3Φ Transformer

$$V_{A1-n} = \left(\frac{N_1}{N_2} \right) V_{A2-B2}$$



For a positive sequence,

$$\begin{aligned} V_{A2-B2} &= V_{A2-n} - V_{B2-n} \\ &= V_{A2-n} - V_{A2-n} \angle -120^\circ \\ &= \sqrt{3} V_{A2-n} \angle 30^\circ \end{aligned}$$

$$V_{A1-n} = \left(\frac{N_1}{N_2} \right) V_{A2-B2} = \boxed{\left(\frac{N_1}{N_2} \right) \sqrt{3}} V_{A2-n} \angle 30^\circ$$

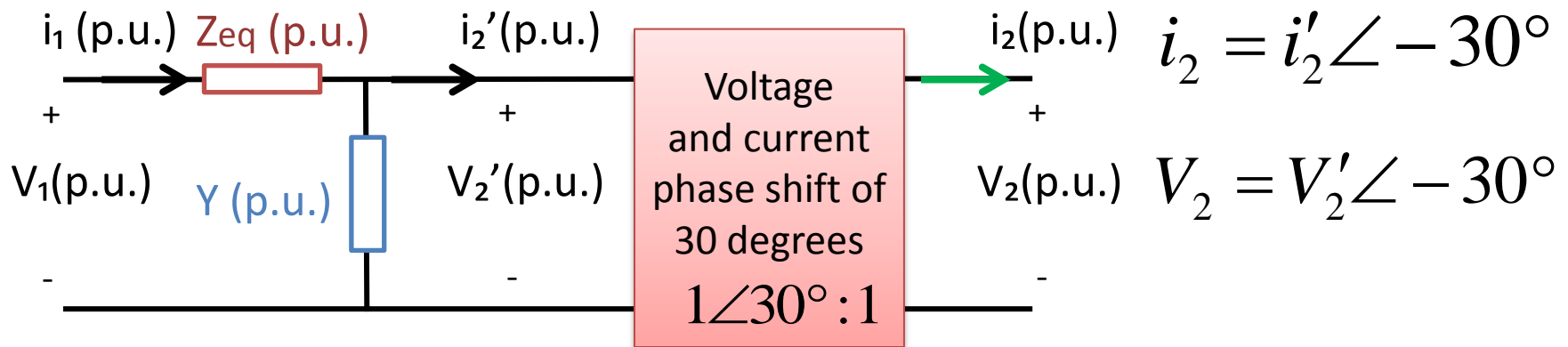
This value becomes the given turn ratio of the transformer.

The line-to-neutral voltage at the primary side leads the line-to neutral voltage at the secondary side by 30 degrees. The same is true for the current.

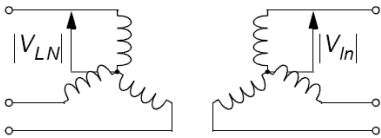
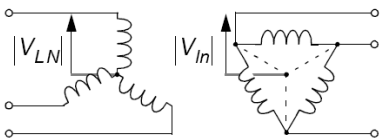
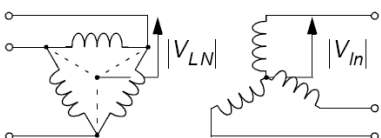
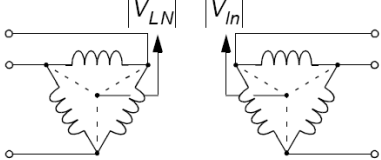
Y-Δ 3Φ Transformer: Per Phase Model

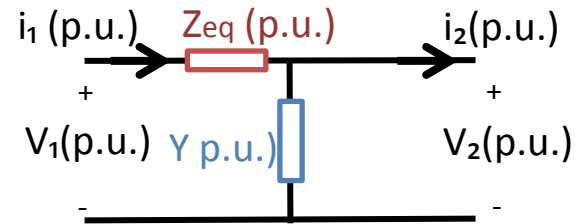
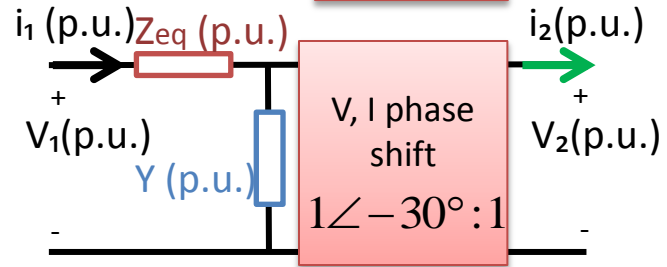
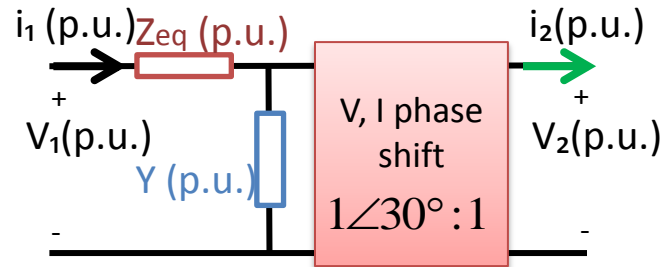
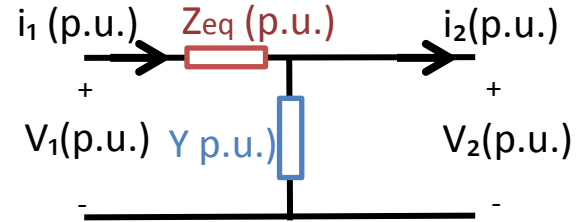
For a positive sequence voltage source,

$$V_{A1-n} = \left(\frac{N_1}{N_2} \right) \sqrt{3} V_{A2-n} \angle 30^\circ \quad \Rightarrow \quad \angle V_{A1-n} : \angle V_{A2-n} = 1 \angle 30^\circ : 1$$

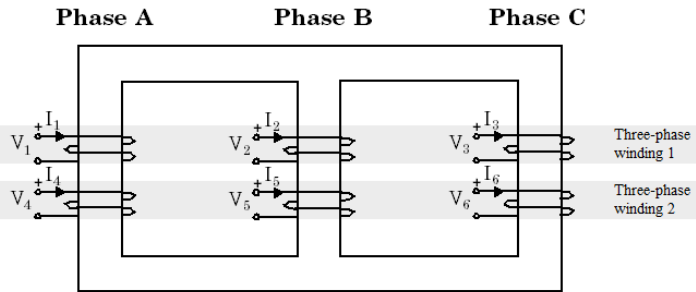


3 Φ Transformer Per Unit Model

designation	winding connection
Yy	
Yd	
Dy	
Dd	

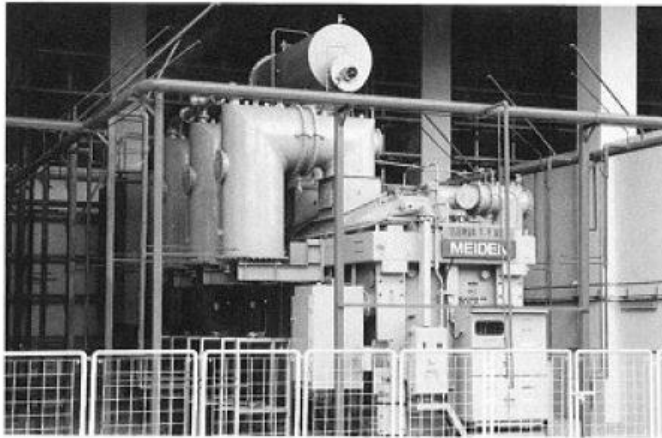


Three-Phase Transformer



Source: www.mathworks.com

230 kV, 150 MVA oil-immersed forced-air cooled transformer
at Kampong Java substation, PowerGrid, Singapore



Source: <http://www.meidensg.com.sg>



Source:

<http://www.youtube.com/watch?v=9Y958Vc5ohI&feature=BFa&list=PLC7AEFD9223780&lf=relist>

5.5 MVA ABB and 10 MVA GE Substation Transformer Nameplate

Source:
<http://www.tucsontransformer.com>

GENERAL ELECTRIC

TRANSFORMER

NO. 1 2571618 CLASS OA THREE-PHASE 60 HERTZ

VOLTS 22900 - 6900Y/3984
 KVA RATING 5500 CONTINUOUS 65 CRISE SELF COOLED
 KVA RATING 17500 CONTINUOUS 65 CRISE FUTURE FORCED AIR

IMPEDANCE VOLTS 5.77 % 22900 - 6900 VOLTS AT 10000 KVA

H V WINDING CONNECTIONS			L V WINDING CONNECTIONS			BASIC IMPULSE INSULATION LEVELS	
VOLTS	AMP 10000 KVA	DIAL POS	VOLTS	AMP 10000 KVA	KVA	ITEM	KV
24100	240	1	L-L	3984	837	H ₁ H ₂ H ₃	150
23500	246	2	L-N			X ₀ X ₁ X ₂ X ₃	95
22900	252	3					
22300	259	4					
21700	266	5					

HV AND LV WINDINGS ALUMINUM

LIQUID LEVEL BELOW TOP SURFACE OF HIGHEST POINT OF MANHOLE FLANGE AT 5 C IS 15 INCHES.
 LIQUID LEVEL CHANGES .86 INCH PER 10 C CHANGE IN LIQUID TEMPERATURE.
 MAXIMUM OPERATING PRESSURES OF LIQUID PRESERVATION SYSTEM 7.5 LBS POSITIVE TO 5 LBS NEGATIVE.
 TANK SUITABLE FOR 14.7 LBS VACUUM FILLING.

APPROX. WEIGHTS IN POUNDS
 TOTAL 46000
 UNTANKING 21500
 TANK AND FITTINGS 1400
 10C OIL 1720 GAL 12900

NP 176B7793

CAUTION BEFORE INSTALLING OR OPERATING READ INSTRUCTIONS

ROME, GEORGIA MADE IN U. S. A.

ABB

SMALL POWER TRANSFORMER
SOUTH BOSTON, VA.

VOLTS		THREE PHASE TYPE RSL	FULL LOAD KVA
HV	4160-	OIL	5500 DA
		INSULATED	65 °C RISE
		UNIT SUBSTATION TRANSFORMER	60 HERTZ
LV	6600Y/3810 X 3300Y/1905 X 4160 X 2080	CLASS OA	GALLONS LIQUID 1643
		INSULATUR INSULATION	MADE IN U.S.A.

INSTRUCTION BOOK PC-1002 SERIAL HBB4411-002T MANUFACTURE DATE 8/98

IMPEDANCE	% AT 5500 KVA	VOLTS
8.84	% AT 5500 KVA	4160 TO 6600Y VOLTS.
8.99	% AT 5500 KVA	4160 TO 3300Y VOLTS.
7.19	% AT 5500 KVA	4160 TO 4160 VOLTS.
7.23	% AT 5500 KVA	4160 TO 2080 VOLTS.

APPROX. WEIGHT IN LBS. 15824 CASE 10456 LIQUID 12322 TOTAL 38602

CONNECTIONS									
WINDING	VOLTS	5500 KVA AMP	TAP CHANGER POS.	CONNECTS	POS.	SP SWITCH D- CONNECTS	POS.	DY SWITCH D- CONNECTS	POS.
HIGH VOLTAGE DELTA	4575	693.9	1	4 10 5					
	4325	727.0	2	3 10 5					
	4160	763.3	3	2 10 5					
	3950	803.5	4	2 10 5					
	3810	833.4	5	2 10 7					
LOW VOLTAGE WYE	6600	481.1			S	A10 TO A11 B10 TO B11 C10 TO C11	3	A9 TO C9 C9 TO B9 B9 TO A9	
	3300	962.2			P	A9 TO A11 B9 TO B11 C9 TO C11	2	A9 TO C9 C9 TO B9 B9 TO A9	
	2080	1520.6			P	A9 TO A11 B9 TO B11 C9 TO C11	1	A12 TO B9 B12 TO C9 C12 TO A9	

D- SP SWITCH CONTACT D- DY SWITCH CONTACT

"DRAIN VALVE STOP"

HV WINDING IS COPPER, LV WINDING IS COPPER
 THE TRANSFORMER MUST NOT BE ENERGIZED FROM ANY VOLTAGE SOURCE WHEN TAP CHANGER OR SP SWITCH OR DY SWITCH IS OPERATED.
 THE OPERATING PRESSURE IS LIMITED BY THE SEALED-AIR FLUID PRESERVATION SYSTEM, WHEN PROVIDED, TO 0.5 LBS. PER SQUARE INCH POSITIVE AND 0.5 LBS. PER SQUARE INCH NEGATIVE.
 THE TRANSFORMER TANK IS DESIGNED TO WITHSTAND A POSITIVE OR NEGATIVE PRESSURE OF 15 LBS. PER SQUARE INCH.
 THE 25°C LIQUID LEVEL IS 12.95 INCHES BELOW HIGHEST MANHOLE FLANGE.
 LIQUID LEVEL CHANGES .57 INCH FOR EACH 10 C CHANGE IN AVERAGE LIQUID TEMPERATURE SUITABLE FOR OPERATION AT OR BELOW 3300 FEET ALTITUDE AT NAMEPLATE RATING.
 AT THE TIME OF SHIPMENT THIS UNIT CONTAINED LESS THAN 1 PPM OF PCBs.

Per Unit Quantity: 3Φ

Base Value: 3Φ

Steps of Calculation: 3Φ

THREE PHASE PER UNIT ANALYSIS

Recall: Per Unit Quantity

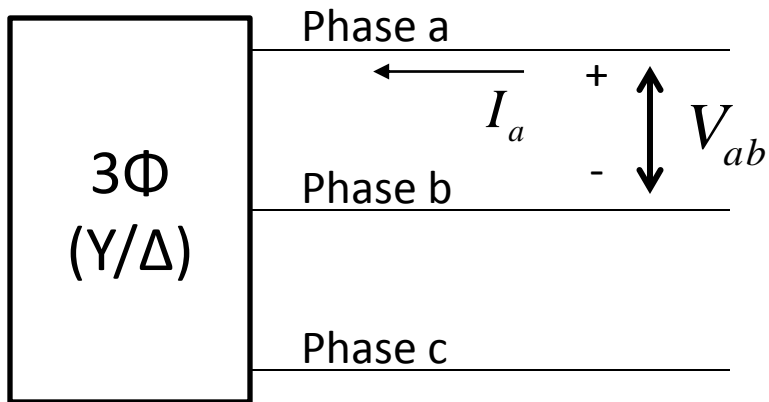
- The per unit quantity of voltage, current, power and impedance is found from dividing the actual quantity by a base value of that quantity.

$$\text{per – unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

- Per unit value is denoted by ‘p.u.’.
- The base value quantity typically follows transformer rating.
- The per unit values are the same irrespective of the sides of the transformer.

Per Unit Analysis in 3 Φ Circuit

- Recall that in three phase circuit:
 - Voltage is given as line-to-line voltage.
 - Current is given as line current.
 - Apparent power is given as three-phase power.



$$|S_{3\Phi}| = \sqrt{3} |V_{\text{Line-To-Line}}| |I_{\text{Line}}|$$

Base Value Selection: 3Φ Case

- Base values are **real numbers**, denote by subscript 'B'.
- Voltage base values follow transformer voltage ratings.
 - The voltage ratios are given as line-to-line voltage at both ends. This means that it already incorporates effects of the Y or Δ configurations.

$$V_B^{line-to-neutral} = \frac{V_B^{line-to-line}}{\sqrt{3}}$$

- Only **single** base complex power $S_B^{3\Phi}$ in the system.
- The base value of power is used to normalize the quantity. Thus, the base values of real power, reactive power, and complex power are all the same real number.

$$P_B^{3\Phi} = Q_B^{3\Phi} = S_B^{3\Phi}$$

Base Value Selection: 3Φ Case

- Current base values are calculated from the base power and base voltage.

$$I_B = \frac{S_B^{3\Phi}}{\sqrt{3}V_B^{line-to-line}} = \frac{S_B^{1\Phi}}{V_B^{line-to-neutral}}$$

- Impedance base values (same value for impedance, resistance, or reactance) are calculated from voltage and current.

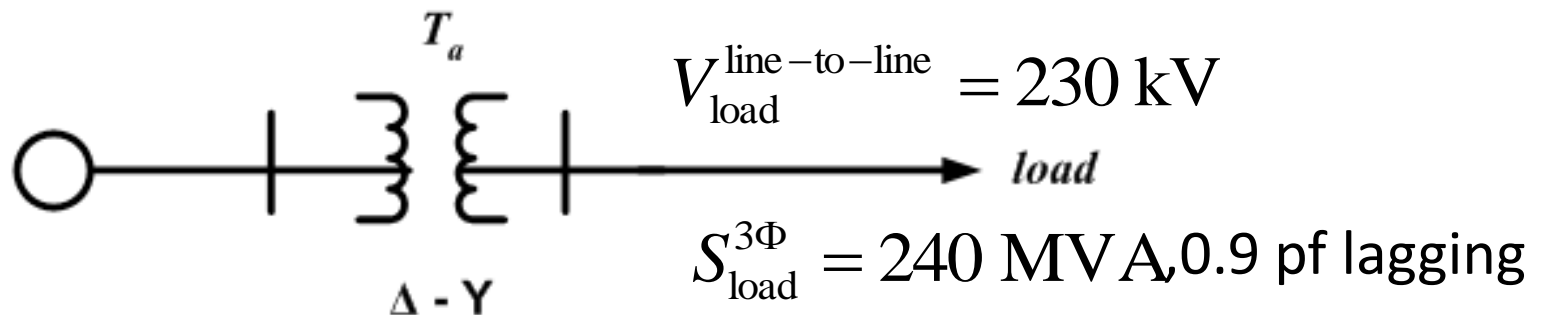
$$Z_B = \frac{V_B^{line-to-neutral}}{I_B} = \frac{\left(V_B^{line-to-neutral}\right)^2}{S_B^{1\Phi}} = \frac{\left(V_B^{line-to-line}\right)^2}{S_B^{3\Phi}}$$

Steps of Calculation: 3 Φ Case

1. Choose $S_B^{3\Phi}$ for the system.
2. Select $V_B^{line-to-neutral}$ or $V_B^{line-to-line}$ for different zones.
3. Calculate Z_B for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.

Example 1: Δ -Y 3 Φ Transformer

- Three-phase generator rated 300 MVA, 23 kV supplying a system load of 240 MVA, 0.9 power factor lagging at 230 kV through a 330 MVA 23 Δ / 230Y kV **step-up** transformer of leakage reactance 11%.
- Use **base values at the load of 100 MVA and 230 kV**, find the current supplied by the generator. Use *load voltage for an angle reference*.



Example 1: Base Values

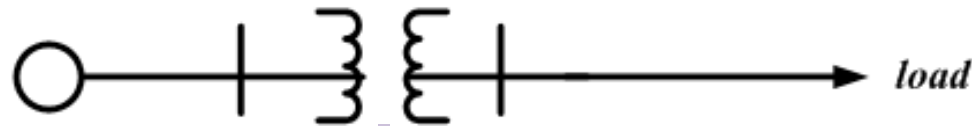
$$S_B^{3\Phi} = 100 \text{ MVA}$$

$$330 \text{ MVA}$$

$$23\Delta / 230Y \text{ kV}$$

$$X_{T1} = 0.11 \text{ p.u.}$$

$$T_a$$



$$V_{B1}^{\text{line-to-line}} = 23 \text{ kV}$$

$$V_{B1}^{\text{line-to-neutral}} = \frac{23}{\sqrt{3}} \text{ kV}$$

$$Z_{B1} = \frac{(V_{B1}^{\text{line-to-line}})^2}{S_B^{3\Phi}} = \frac{(23 \times 10^3)^2}{100 \times 10^6} = 5.29 \Omega$$

$\Delta - Y$

$$V_{B2}^{\text{line-to-line}} = 230 \text{ kV}$$

$$V_{B2}^{\text{line-to-neutral}} = \frac{230}{\sqrt{3}} \text{ kV}$$

$$Z_{B2} = \frac{(V_{B2}^{\text{line-to-line}})^2}{S_B^{3\Phi}} = \frac{(230 \times 10^3)^2}{100 \times 10^6} = 529 \Omega$$

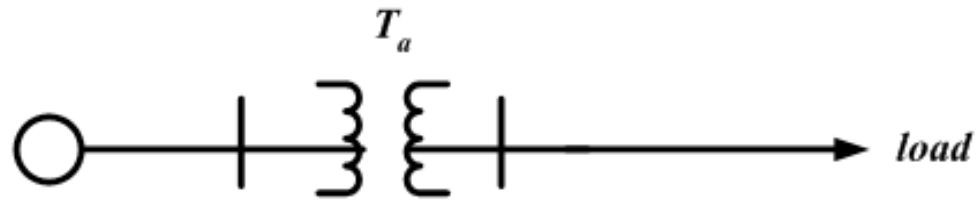
$$I_{B1} = \frac{S_B^{3\Phi}}{\sqrt{3} V_{B1}^{\text{line-to-line}}} = \frac{100 \times 10^6}{\sqrt{3} \times 23 \times 10^3} = 2510.2 \text{ A}$$

$$I_{B2} = \frac{S_B^{3\Phi}}{\sqrt{3} V_{B2}^{\text{line-to-line}}} = \frac{100 \times 10^6}{\sqrt{3} \times 230 \times 10^3} = 251.02 \text{ A}$$

Example 1: Transformer Reactance

$$S_B^{3\Phi} = 100 \text{ MVA}$$

$$\begin{aligned} &330 \text{ MVA} \\ &23\Delta / 230Y \text{ kV} \\ &X_{T1} = 0.11 \text{ p.u.} \end{aligned}$$



$$V_{B1}^{\text{line-to-line}} = 23 \text{ kV}$$

$$V_{B2}^{\text{line-to-line}} = 230 \text{ kV}$$

$$Z_{B1} = 5.29 \Omega \quad I_{B1} = 2510.2 \text{ A}$$

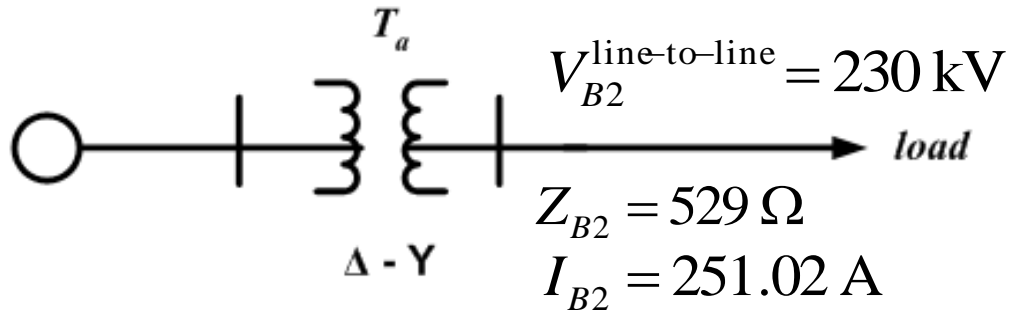
$$Z_{B2} = 529 \Omega \quad I_{B2} = 251.02 \text{ A}$$

$$X_{T,\text{p.u.}}^{\text{new}} = \frac{X_{\text{p.u.}}^{\text{old}} X_B^{\text{old}}}{X_B^{\text{new}}} = \frac{0.11 \left(\frac{(230 \times 10^3)^2}{330 \times 10^6} \right)}{529} = 0.03333 \text{ p.u.}$$

$$X_{T,\text{p.u.}}^{\text{new}} = \frac{X_{\text{p.u.}}^{\text{old}} X_B^{\text{old}}}{X_B^{\text{new}}} = \frac{0.11 \left(\frac{(23 \times 10^3)^2}{330 \times 10^6} \right)}{5.29} = 0.03333 \text{ p.u.}$$

Example 1: Load Voltage and Current

$$S_B^{3\Phi} = 100 \text{ MVA}$$



At the load, use load voltage as reference.

$$V_{\text{load}}^{\text{line-to-line}} = 230 \angle 0^\circ \text{ kV}$$

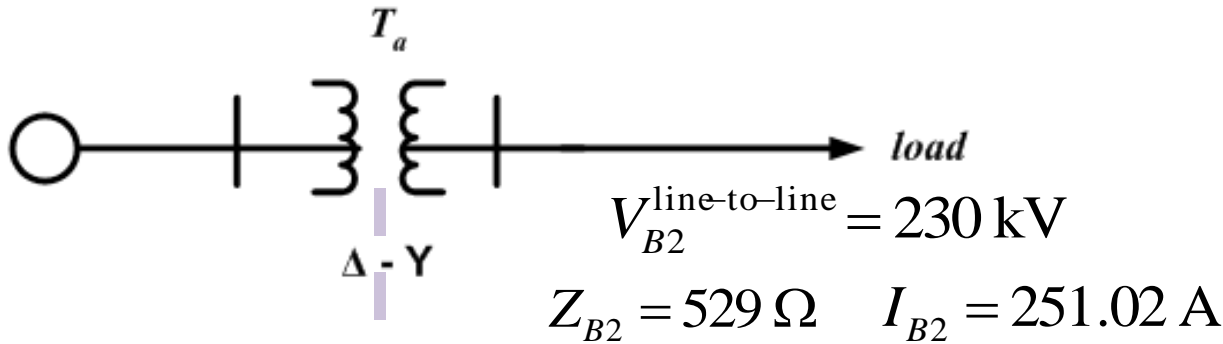
Load current magnitude is found from the three-phase apparent power and line-to-line voltage.

$$|I_{\text{load}}| = \frac{|S_{\text{load}}^{3\Phi}|}{\sqrt{3} |V_{\text{load}}^{\text{line-to-line}}|} = \frac{240 \times 10^6}{\sqrt{3} \times 230 \times 10^3} = 602.45 \text{ A}$$

Load current angle can be found from power factor. Note that the reference angle of load voltage is zero degree.

$$\angle I_{\text{load}} = \angle -\cos^{-1}(0.9)^\circ = -25.842^\circ$$

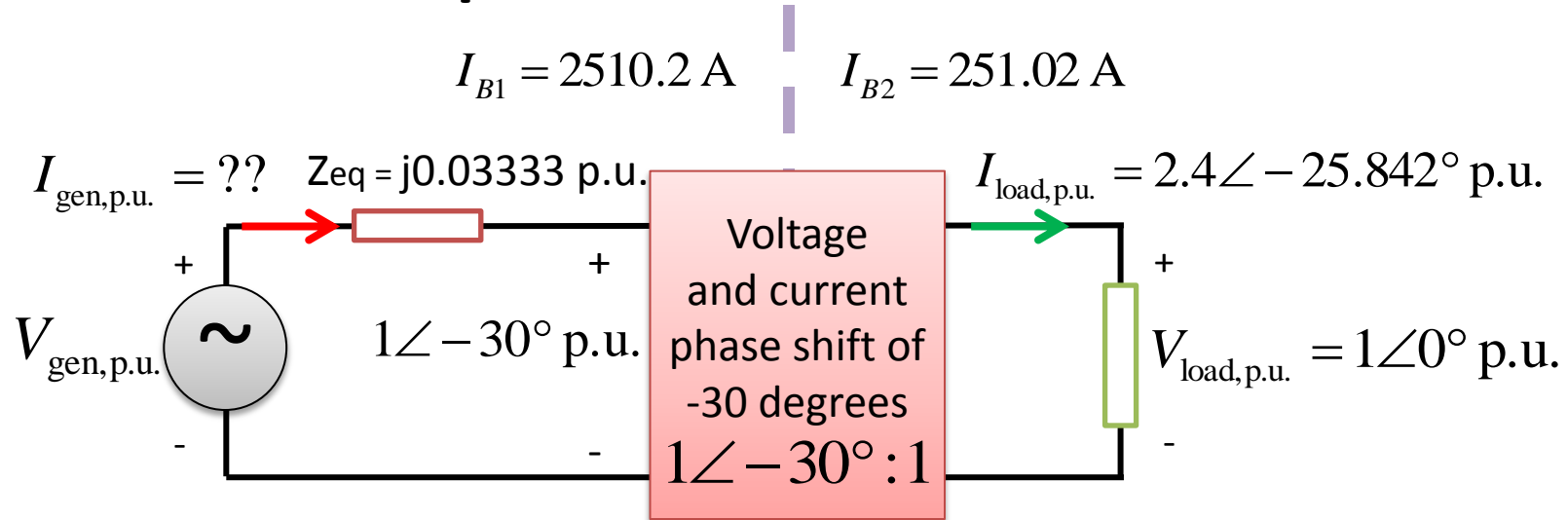
Example 1: Per Unit Values



$$V_{\text{load,p.u.}} = \frac{V_{\text{load}}^{\text{line-to-line}}}{V_{B2}^{\text{line-to-line}}} = \frac{230 \angle 0^\circ \text{ kV}}{230 \text{ kV}} = 1 \angle 0^\circ \text{ p.u.}$$

$$I_{\text{load,p.u.}} = \frac{I_{\text{load}}}{I_{B2}} = \frac{602.45 \angle -25.842^\circ \text{ A}}{251.02 \text{ A}} = 2.4 \angle -25.842^\circ \text{ p.u.}$$

Example 1: Per Unit Circuit



$$\frac{I_{\text{gen,p.u.}}}{I_{\text{load,p.u.}}} = \frac{1 \angle -30^\circ}{1}$$

$$I_{\text{gen,p.u.}} = I_{\text{load,p.u.}} \angle -30^\circ = 2.4 \angle -55.842^\circ$$

$$I_{\text{gen}} = I_{\text{gen,p.u.}} \times I_{B1} = 2.4 \angle -55.842^\circ \times 2510.2 = 6024.5 \angle -55.842^\circ \text{ A}$$

Can you find the line-to-line voltage supplied by the generator?

$$\text{Ans: } V_{\text{gen}}^{\text{line-to-line}} = 23.86 \angle -26.02 \text{ kV}$$

Summary

- Per unit system helps to eliminate transformer in the circuit analysis.
- Per unit value is found by normalize the actual value by base value.
- We can divide circuits into zones according to transformer voltage ratings.
- Choose only single base power. Voltage, current, and impedance base value is calculated for each zone.
- Actual value is found from multiplying the per unit value to its corresponding base value in its zone.
- Three-phase transformer Y- Δ and Δ -Y connections introduces phase shift of 30 and -30 degrees correspondingly.

Subsequent Lectures

1. **Introduction** – Electrical energy systems
2. **AC systems basics** - Active, Reactive and Apparent Power; Concept of Power factor, lagging, leading and unity power factor operation;
3. **Single- and three-phase power system**; Star and Delta connection; Relationship between phase and line quantities; per-unit representation.
4. **Generation, Transmission and Distribution Network** - Power system structure; Generation, Transmission and Distribution; Transmission line modeling; Complex power transmission; Transformer, Principle of operation of synchronous and asynchronous machines; Applications.
5. **Renewable Energy Sources** - Sustainable and clean energy sources; Solar Photovoltaic, Wind, Hydro, Fuel-cell
6. **Distributed Generation** - Concept of distributed energy generation and utility interfacing; Smart Grid
7. **Energy storage** - Concept of energy storage; Batteries, Super-capacitors, Fly-wheels
8. **Singapore Electricity Network** - Singapore power system network, Electricity system unbundling; **Electric energy market** structure and operation; Cost of electricity

1st Mid-term Test

- March 5th 5:15pm to 6:00pm @LT 4
- Materials (Lecture 1-5, Tutorials 1-3):
 - AC circuit analysis
 - Apparent power, power factor correction
 - Three-phase voltage and current
 - Three-phase complex power
- Format:
 - 4 questions, closed book.
 - 45 minutes.