Adaptive Visual Servoing Scheme free of Image Velocity Measurement for Uncertain Robot Manipulators

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Abstract

This work addresses the visual tracking problem of robot manipulators with non-negligible dynamics using a fixed camera, when the camera-robot system parameters are uncertain. An adaptive strategy is developed for visual servoing systems based on the image-based look-and-move structure to allow the tracking of a 2D reference trajectory, without using image velocity measurements. The adaptive visual servoing problem free of image velocity information is formulated as a relative degree two MIMO adaptive control problem. As a solution, we employ a recently proposed Lyapunov/passivity-based adaptive control scheme based on SDU factorization method. From a cascade control strategy, the resulting online camera calibration scheme is combined with a direct adaptive motion controller for the robot manipulator, which takes into account its uncertain nonlinear dynamics. The overall stability of the adaptive visual servoing system can be proved thanks to the explicit Lyapunov-like functions of both adaptation schemes.

Key words: Adaptive control, Cascade control, Lyapunov stability, Robot vision, Robotic manipulators.

1 Introduction

For many years, the visual feedback has been explored in several works as a control-theoretical issue and the problem of controlling robotic systems by means of vision sensing has still received outstanding attention in the control system literature [1, 2]. In turn, it is well known that the measurement of the image velocity is impaired by noisy image data [3] and the kinematics and dynamics parameters of the robot arm are modified when its end-effector is handling different tools with unknown lengths [4]. This motivates the design of an alternative adaptive scheme free of image velocity measurement, which considers the presence of parametric uncertainty in the camera and robot models. In this context, two direct adaptive camera calibration schemes were proposed in [5, 6] with global asymptotic stability properties. However, the authors assume exact knowledge of the robot kinematics and dynamics. In turn, an adaptive control strategy for uncertain robot manipulators was presented in [7], where the robot kinematics and camera parameters of the Jacobian matrix were indirectly updated.

In our paper, we propose a solution for the adaptive visual tracking problem of robot manipulators using a fixed camera, when both camera calibration and robot parameters are uncertain. The strategy is developed for 2D visual servoing systems based on the image-based look-and-move structure. In order to solve the MIMO adaptive control problem related to the adaptive camera calibration scheme, without using image velocity information, an uncertain linear plant with relative degree two has to be considered. A recently proposed model reference adaptive control (MRAC) for relative degree two MIMO systems [8] using SDU factorization method [9] is considered. This new solution is Lyapunov/passivity-based in the sense that an explicit Lyapunov function exists for the complete state of the adaptive system. The importance of having an explicit Lyapunov for the adaptive camera calibration scheme is that it can be easily combined with well-known adaptive motion controller for the robot manipulator, which takes into account its uncertain nonlinear dynamics, leading to an overall globally stable adaptive system.
2 Robot System Model

Consider the visual tracking problem of a 2D reference trajectory with a robot manipulator using a fixed and uncalibrated camera, with optical axis perpendicular to the robot working plane. Let \( y_c \in \mathbb{R}^2 \) be the centroid position of the image feature fixed on the robot end-effector and \( \hat{y}_{cd} \in \mathbb{R}^2 \) be the desired time-varying trajectory, both expressed in the image frame. The control goal can be described by:

\[
y_c \rightarrow \hat{y}_{cd}(t), \quad e_c = y_c - \hat{y}_{cd}(t) \rightarrow 0,
\]

where \( e_c \in \mathbb{R}^2 \) is the image tracking error. From the perspective projection model of a pin-hole camera, the Cartesian space can be related to the image space by the following transformation:

\[
y_c = \frac{f}{z_0} \begin{bmatrix} \alpha_1 \cos(\phi) & -\alpha_1 \sin(\phi) \\ \alpha_2 \sin(\phi) & \alpha_2 \cos(\phi) \end{bmatrix} y + y_0,
\]

where \( y \in \mathbb{R}^2 \) is the end-effector position with respect to the robot base, expressed in the base frame, obtained from the forward kinematics map \( y = h(q) \), \( q \in \mathbb{R}^2 \) is the vector of manipulator joint angles, \( y_0 \in \mathbb{R}^2 \) is a bias vector, \( K_p \in \mathbb{R}^{2 \times 2} \) is the camera/workspace transformation matrix, \( z_0 \in \mathbb{R}^+ \) is the total depth between the camera focal point and the working plane, \( f \in \mathbb{R}^+ \) is focal length of the camera lens (in general, \( z_0 \gg f \)), \( \alpha_1, \alpha_2 \) are positive scaling factors of the camera (in pixel per millimeter) and \( \phi \in (-\pi, \pi) \) is the misalignment angle between the camera and robot frames. Here, we assume that the camera and robot base frames are aligned only with respect to the z-axis.

The end-effector velocity \( \dot{y} \in \mathbb{R}^2 \) can be related with the joint velocity vector \( \dot{q} \in \mathbb{R}^2 \) by

\[
\dot{y} = J(q) \dot{q} = W(q, \dot{q}) b,
\]

where \( J(q) = \frac{\partial h(q)}{\partial q} \in \mathbb{R}^{2 \times 2} \) is the analytical Jacobian. Note that, the second term of (3) can be linearly parameterized by \( W(q, \dot{q}) b \), where \( b \in \mathbb{R}^n \) denotes the constant kinematic parameters and \( W \in \mathbb{R}^{2 \times n} \) is the kinematic regressor matrix.

Now, we recall that the nonlinear dynamic model of the robot manipulator can be expressed in generalized coordinates by [10]

\[
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = Y(q, \dot{q}, \ddot{q}) a = \tau,
\]

where \( M(q) \in \mathbb{R}^{2 \times 2} \) is the manipulator inertia matrix, \( C(q, \dot{q}) \dot{q} \in \mathbb{R}^2 \) represents the centrifugal and Coriolis torques, \( g(q) \in \mathbb{R}^2 \) represents the gravity torques, and \( \tau \in \mathbb{R}^2 \) is the vector of applied joint torques. It is well known that the left-hand side of (4) can be linearly parameterized by \( Y(q, \dot{q}, \ddot{q}) a \), where \( a \in \mathbb{R}^m \) denotes the constant dynamic parameters and \( Y \in \mathbb{R}^{2 \times m} \) is the dynamic regressor matrix. It is worth mentioning the following properties of \( M(q) \) and \( C(q, \dot{q}) \): (i) \( M(q) \) is a symmetric and positive definite matrix for all \( q \in \mathbb{R}^2 \); (ii) for a particular choice of the matrix \( C(q, \dot{q}) \) the following relation holds \( w^T (M - 2C) w = 0 \) for all \( w \in \mathbb{R}^2 \).

3 Visual Servo Control Strategy

In this section, we describe a kinematic control strategy using the visual servoing approach. Since \( z_0 \) is constant in the planar case, the Cartesian control problem can be described from (2) as:

\[
\hat{y}_c = K_p v_k,
\]

where \( v_k = \hat{y} \) with kinematic control law \( \dot{q} = J^{-1}(q) v_k \). Then, the visual tracking problem is formulated as designing \( v_k(t) \in \mathbb{R}^2 \) in (5) so that the image tracking error \( e_c(t) \) tends asymptotically to zero as \( t \) tends to infinity.

In order to achieve this aim, two basic assumptions are considered: (A1) The desired image trajectory \( \hat{y}_{cd} \) is defined within the robot workspace, and the desired image velocity \( \hat{y}_{cd} \) is known and bounded; (A2) Manipulator motions are away from Jacobian singularities. From these assumptions, the occlusion related problems can be avoided and the inverse Jacobian always exist. In this work, we consider the case where (i) the kinematic and dynamic parameters of the robot arm are uncertain and (ii) the camera is not calibrated. Hence, the parameters \( b \) and \( a \), in (3) and (4), as well as the camera/workspace transformation \( K_p \) in (5) are both unknown.

4 Cascade Strategy

Here, the key idea is to introduce a cascade control strategy [11] to solve the visual tracking problem for a robot manipulator with non-negligible dynamics. To this end, we can assume that there exists a control law \( \tau = F(q, \dot{q}, q_m, \dot{q}_m, \ddot{q}_m) \) which guarantees the control goal defined by

\[
q \rightarrow q_m(t), \quad e = q - q_m(t) \rightarrow 0,
\]

where \( e \in \mathbb{R}^2 \) is the angular position error and \( q_m \in \mathbb{R}^2 \) is the desired trajectory expressed in the joint space. Now, from Figure 1, suppose we can define the desired trajectory \( q_m \) and its derivatives \( \dot{q}_m, \ddot{q}_m \) in terms of a Cartesian control signal \( v \), such that we have (5) except for a vanishing term, that is,

\[
\hat{y}_c = K_p [v + J(q) G(s) e],
\]
where \( G(\cdot) \) denotes a linear operator, possibly non-causal, with \( s \) being the differential operator.

Recently, under the formulation (7), adaptive visual servoing schemes were proposed in [11] [12] [13] to deal with the uncertainties in the camera calibration parameters, namely the scaling factors and camera orientation angle with respect to the robot frame. These schemes could also include the uncertainties of the robot dynamic parameters. However, the robot motion adaptive control involves a regressor matrix that depends on the image velocity. Indeed, in the adaptive scheme presented in [11], a linear visual servoing system with relative degree one from \( v \) to \( y_e \) is obtained by defining \( \ddot{q}_m = J^{-1}(q) v + \lambda e \).

As a result, the computation of \( \ddot{q}_m \) requires on \( \dot{v} \) and, consequently, the time-derivative of \( y_e \).

Here, the main idea to avoid the need for measuring the velocity in the image space is to consider a filtered version of \( q_m \) proposed in [11], that is,

\[
\ddot{q}_m = J^{-1}(q) H^{-1}(s) [v + \lambda J(q) e],
\]

where \( H(s) = (s + \lambda) I \) is a first order Hurwitz polynomial, such that the resulting visual servoing system has relative degree \( n^* = 2 \)

\[
\ddot{y}_c = K_p [v + H(s) J(q) G(s) e],
\]

as it will be presented later on in Section 5.1, and therefore it can be controlled using the adaptive control scheme proposed in this paper. Furthermore, for the stability analysis of the proposed cascade scheme, the passivity framework derives simple rules to describe combinations of subsystems expressed in a Lyapunov formalism. For cascaded passive systems the following general result can be stated [8].

**Theorem 1** Consider the following interconnected systems, where \( \Sigma_1 \) is the driven system and \( \Sigma_2 \) is the driving system:

\[
\begin{align*}
\Sigma_1 : & \begin{cases} 
\dot{x}_1 = f_1(x, t) + g_1(x, t) y_2, \\
\quad y_1 = h_1(x_1),
\end{cases} \\
\Sigma_2 : & \begin{cases} 
\dot{x}_2 = f_2(x, t) + u_2, \\
\quad y_2 = h_2(x_2),
\end{cases}
\end{align*}
\]

where \( f_1, f_2 \) are piecewise continuous in \( t \) and locally Lipschitz in \( x \) for all \( t \geq 0 \) and \( x \in \mathbb{D} \), and \( \mathbb{D} \subset \mathbb{R}^n \) is a domain that contains the origin \( x = 0 \), \( h_1, g_1, h_2, g_2 \) are continuous. Assume that \( \| g_1(x, t) \| \leq c, \) for some \( c > 0 \). If system \( \Sigma_1 \) is output strictly passive from \( y_2 \rightarrow y_1 \) with positive definite storage function \( V_1(x_1) \) such that

\[
\dot{V}_1 \leq -\lambda_1 \| y_1 \|^2 + c_1 y_2^T y_1, \quad \lambda_1 > 0
\]

and system \( \Sigma_2 \) is output strictly passive from \( u_2 \rightarrow y_2 \) with positive definite storage function \( V_2(x_2) \) such that

\[
\dot{V}_2 \leq -\lambda_2 \| y_2 \|^2 + c_2 u_2^T y_2, \quad \lambda_2 > 0.
\]

Then, for \( u_2 = 0 \) implies that: (i) \( x_1, x_2 \in \mathcal{L}_\infty \); (ii) \( \lim_{t \to \infty} y_1(t) = 0, \lim_{t \to \infty} y_2(t) = 0 \) (for a proof see [8]).

5 Adaptive Robot Control

Considering the presence of parametric uncertainty in the robot kinematic model (3), the estimated end-effector velocity \( \hat{y} \) can be expressed as

\[
\hat{y} = J(q) \hat{q} = W(q, \hat{q}) \hat{b},
\]

where \( J(q) \in \mathbb{R}^{2 \times 2} \) is the approximate analytical Jacobian and \( \hat{b} \in \mathbb{R}^n \) denotes a set of estimated kinematic parameters. It is known from [14] that the linear parameterization models (3) and (10) can be used for online parameter estimation, provided that \( \hat{y} \) and \( W \) are measured from the system signals. In order to avoid the need of measuring the end-effector velocity \( \dot{y} \), we use a low-pass filter \( F(s) = \lambda_p / (s + \lambda_p) \) with \( \lambda_p > 0 \), such as:

\[
y_p = F(s) \hat{y} = W_p \hat{b}, \quad W_p = F(s) W(q, \hat{q}),
\]

where \( y_p \in \mathbb{R}^2 \) is the filtered output of the end-effector velocity and \( W_p \) is a filtered regressor matrix. Then, by defining the estimation error \( e \in \mathbb{R}^2 \) as \( e := \hat{y}_p - y_p \), the gradient-type adaptive law for updating \( \hat{b} \) is given by

\[
\dot{\hat{b}} = -\Gamma_k W_p^T e, \quad \Gamma_k = \Gamma_k^T > 0,
\]

where \( \Gamma_k \) is the adaptive gain matrix. A simple proof of the stability for this estimation algorithm can be found in [14]. Hence, by using an indirect adaptive scheme given by (12) the estimation of \( J \) can be embedded in the proposed adaptive solution, provided that \( y_p \) and \( W_p \) can be measured from the systems signals.

Now, considering that the robot dynamic model (4) is also uncertain, we will show that the adaptive control design for the robot system can be derived by simply cascading the proposed adaptive visual servoing scheme with the Slotine-Li adaptive scheme [14]. Hence, we can define the following signals in joint space:

\[
\dot{q}_r := \ddot{q}_m - \lambda e, \quad \sigma := \dot{q} - \dot{q}_r = \dot{e} + \lambda e.
\]
where \( \sigma \in \mathbb{R}^2 \) is the virtual error. We also can define the regressor matrix \( Y(q, \dot{q}, \dot{\hat{q}}, \ddot{\hat{q}}) \) by setting
\[
\dot{M}(q) \dot{\hat{q}}_r + \dot{\hat{C}}(q, \dot{q}) \dot{\hat{q}}_r + \dot{\hat{g}}(q) = Y(q, \dot{q}, \dot{\hat{q}}_r, \ddot{\hat{q}}_r) \dot{\hat{a}},
\]
where \( \dot{a} \) is a constant vector of uncertain parameters to be adapted on-line and \( M, \dot{C}, \dot{g} \) denote the estimated terms of the dynamic model (4). In addition, the control law is given by
\[
\tau = Y(q, \dot{q}, \dot{\hat{q}}_r, \ddot{\hat{q}}_r) \dot{a} - K_D \sigma + u_2,
\]
where \( K_D \) is a positive definite gain matrix, \( u_2 \) is a fictitious external input which drives the closed-loop system and \( \dot{a} \) can be updated using the gradient-type adaptive law
\[
\dot{\hat{a}} = -\Gamma_d Y^T \sigma, \quad \Gamma_d = \Gamma_d^T > 0,
\]
where \( \Gamma_d \) is the adaptation gain matrix. Thus, the stability analysis and passivity properties of the closed-loop system can be established in the following theorem:

**Theorem 2** Consider the robot dynamic model (4), the control law (15) and the adaptation law (16). Then, the map \( u_2 \rightarrow \sigma \) is output strictly passive with positive definite storage function
\[
2V_d(\sigma, \dot{a}) = \sigma^T M(q) \sigma + \dot{a}^T \Gamma_d^{-1} \dot{a},
\]
with \( \dot{a} = \dot{a} - a \). Furthermore, for \( u_2 = 0 \), the following properties hold: (i) all system signals are bounded; (ii) \( \lim_{t \rightarrow -\infty} \sigma(t) = 0 \), which implies that \( \lim_{t \rightarrow -\infty} \dot{\hat{e}}(t) = 0 \) and \( \lim_{t \rightarrow -\infty} e(t) = 0 \) (for a proof see [14]).

### 5.1 Cascade Control Scheme

Now, we can apply the cascade control strategy presented in the previous section. Based on the cascade structure and from (13), we can define
\[
\dot{\hat{q}}_r = \tilde{J}^{-1}(q) H^{-1}(s) [v + \lambda \tilde{J}(q) \dot{\hat{q}}],
\]
such that the motion of the robot end-effector in the image space is governed by
\[
\dot{y}_c = K_p [v + H(s) \tilde{J}(q) \sigma] + \lambda K_p w,
\]
where \( \sigma \) can be considered as a vanishing disturbance since, from Theorem 2, it tends asymptotically to zero and \( w = [\tilde{J}(q) - \tilde{J}(q)] \dot{q} = W(q, \dot{q}) \ddot{b} \) is a bounded disturbance term due to the uncertain kinematics. Thus, the cascade strategy can be obtained by simply setting
\[
\dot{q}_m = \dot{\hat{q}}_r + \lambda \dot{e}, \quad \dot{q}_m = \ddot{\hat{q}}_r + \lambda \ddot{e},
\]
where \( \dot{q}_m \) depends on \( v \) (and not on \( \dot{v} \)) since
\[
\dot{\hat{q}}_r = \tilde{J}^{-1}(q) [v - \tilde{J}(q) \dot{\hat{q}}] + \lambda \sigma.
\]

It is worth noting that the proposed cascade structure can be also performed using a robust robot motion control with similar passivity properties [14].

### 6 Adaptive Visual Servoing

For the control design, we can obtain the following state space realization for (19):
\[
\begin{align*}
\dot{x}_c &= A x_c + B K_p v + B_\sigma \tilde{J}(q) \sigma + B_w w, \\
y_c &= C x_c,
\end{align*}
\]
where \( C = [I \ 0] \),
\[
A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} K_p \\ \lambda K_p \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ \lambda K_p \end{bmatrix}.
\]

Now, let us define the desired trajectory in the image space \( y_{cd} \) by means of a model reference:
\[
y_{cd} = G_m(s) r, \quad G_m(s) = \frac{\lambda_c^2}{(s + \lambda_c)^2} I,
\]
where \( r(t) \) is piecewise continuous and uniformly bounded and \( \lambda_c \) is a free positive parameter. The MRAC approach could lead us to the matching control by output feedback. Here, we are able to adopt a simpler approach by taking into account that the plant is essentially a double integrator, except for the matrix gain \( K_p \). Indeed, we can first solve the problem for a unit (matrix) gain double integrator, by output feedback, determining a control law, say, \( u := K_p v \). Then, the matching control law \( u^* \) for (19) would simply be \( u^* = K_p^{-1} u^* \). Then, we define the control parameterization in terms of \( u \) which can be regarded as a regressor vector which is available from only the output and input signals.

Thus, a model matching control law \( u^* \) is given by:
\[
\begin{align*}
u^* &= -2 \lambda \dot{y}_c - \lambda^2 y_c + \lambda^2 r, \\
\dot{z}_1 &= -\lambda_0 z_1 + u, \\
\dot{z}_2 &= -\lambda_0 z_2 + y_c, \\
\dot{\hat{y}}_c &= z_1 - \lambda_0^2 z_2 + \lambda_0 y_c,
\end{align*}
\]
where \( \lambda_0 > 0 \) is a free parameter. In term of variable \( z = z_1 - \lambda_0^2 z_2 \), the last three equations correspond to a reduced order observer of \( \dot{y}_c \). However, note that \( u \) is not measurable since \( K_p \) is unknown. The model matching control law for \( v \) is \( v^* = K_p \lambda v^* \), which can be written:
\[
v^* = q^T \omega - \frac{2 \lambda}{\lambda(s)} v,
\]
with
\[
\omega = \frac{2\lambda_0 \lambda_2^2}{\Lambda(s)} y_c - (\lambda_2^2 + 2\lambda_c \lambda_0) y_c + \lambda_c^2 r,
\]
where \( \theta^{*T} = K_p^{-1} \) and \( \Lambda(s) = s + \lambda_0 \). Note that, from (25)-(28), \( u^* \) can be rewritten as \( u^* = \xi^{*T} \nu + \xi_1^* r \), where \( \xi^{*T} = [-2\lambda_0 \lambda_2 + \lambda_1^2] - 2\lambda_c \lambda_0^2, \) and \( \nu = (y_c^T \bar{z}_1^T \bar{z}_2^T)^T \).

6.1 Image Error System

Now, we can express the image error system in terms of the augmented state \( z_c = [x_c^T \bar{z}_1^T \bar{z}_2^T]^T \) by combining (22), (26) and (27), and defining \( u = K_p v \):

\[
\dot{z}_c = A_1 z_c + B_1 K_p v + B_{cs} \dot{J}(q) \sigma + B_{cb} w, \quad \text{(31)}
\]
\[
y_c = C_1 z_c, \quad \text{(32)}
\]
where
\[
A_1 = \begin{bmatrix} A & 0 & 0 \\ 0 & -\lambda_0 & 0 \\ C & 0 & -\lambda_0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} B \\ I \\ 0 \end{bmatrix}, \quad B_{cs} = \begin{bmatrix} B_s \\ 0 \\ 0 \end{bmatrix}, \quad B_{cb} = \begin{bmatrix} B_w \\ 0 \\ 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} C & 0 & 0 \end{bmatrix}.
\]

By adding and subtracting \( v^* \) to \( v \), and using the fact that \( v^* \) is a model matching control, we have that

\[
\dot{z}_c = A_m z_c + B_m r + B_m K_p (v-v^*) + B_{cs} \dot{J}(q) \sigma + B_{cb} w, \quad \text{(33)}
\]
\[
y_c = C_m z_c, \quad \text{(34)}
\]

where \( A_m = A_1 + B_1 \xi^{*T}, \) \( B_m = B_1 \xi_1^* \) and \( C_m = C_1 \). Note that, the triple \( \{A_m, B_m, C_m\} \) corresponds to a non-minimal realization of the reference model (24), where the relative degree from \( r \) to \( y_c \) is two and consequently \( C_mB_m = 0 \). Then, we can obtain the error system in terms of the error state \( \bar{z}_c = z_c - z_{cd} \) and the image tracking error \( e_c = y_c - y_{cd} \) as:

\[
\dot{z}_c = A_m \bar{z}_c + B_m K_p (v-v^*) + B_{cs} \dot{J}(q) \sigma + B_{cb} w, \quad \text{(33)}
\]
\[
e_c = C_m \bar{z}_c, \quad \text{(34)}
\]

Thus, defining
\[
\nu = \dot{\nu} - \frac{2\lambda_c}{\Lambda(s)} v \quad \text{(35)}
\]
and considering (29) we have that:

\[
\dot{z}_c = A_m \bar{z}_c + B_m K_p (\dot{\nu} - \theta^{*T} \omega) + B_{cs} \dot{J}(q) \sigma + B_{cb} w, \quad \text{(36)}
\]
\[
e_c = C_m \bar{z}_c. \quad \text{(37)}
\]

We can reduce this problem of multivariable adaptive tracking with relative degree two to relative degree one according to (15) by defining the signals

\[
\hat{v}_f = L^{-1}(s) \nu, \quad \omega_f = L^{-1}(s) \omega, \quad \text{(38)}
\]
with \( L(s) = (s + \lambda_c) I \) and rewriting (33) as

\[
\ddot{z}_c = A_m \bar{z}_c + B_m K_p \hat{v}_f + B_{cs} \dot{J}(q) \sigma + B_{cb} w, \quad \text{(39)}
\]
\[
e_c = C_m \bar{z}_c, \quad \text{(40)}
\]

For simplicity, we introduce the following notation: \( \dot{v}_f = \dot{\hat{v}}_f - \theta^{*T} \omega_f \). Then, performing the following change of variable \( \bar{z}_c = z_c - B_m K_p \hat{v}_f \) we obtain

\[
\ddot{z}_c = A_m \bar{z}_c + B_m K_p \dot{v}_f + B_{cs} \dot{J}(q) \sigma + B_{cb} w, \quad \text{(39)}
\]
\[
e_c = C_m \bar{z}_c, \quad \text{(40)}
\]

where \( B_{m1} = A_m B_m + \lambda_c B_m \). To arrive at the system (36), we have taken into account that \( C_mB_m = 0 \). According to the SDU factorization approach for designing MIMO adaptive control \[9, 8\], there exists a factorization \( K_p = SDU \), where \( S \) is symmetric, \( D \) is diagonal, and \( U \) is unit upper triangular matrices respectively. Such a factorization is non unique and we can show that for any choice of \( G_m(s) \) in (24) there exists a positive definite matrix \( S = S^T \) such that \( \{A_m, B_{m1} S, C_m\} \) is SPR \[9\], that is, there exist positive definite matrices \( P \) and \( Q \) such that

\[
A_m^T P + P A_m = -2Q, \quad S B_m^T P = C_m. \quad \text{(41)}
\]

Thus, the error equation can be written into a new form:

\[
\dot{\bar{z}}_c = A_m \bar{z}_c + B_m S D(\dot{\nu}_f - \bar{v}_f) + B_{cs} \dot{J}(q) \sigma + B_{cb} w, \quad \text{(42)}
\]
\[
e_c = C_m \bar{z}_c, \quad \text{(43)}
\]

where \( \bar{v}_f = U \theta^{*T} \omega_f + (I - U) \dot{\nu}_f \). The key feature of (42) is that the diagonal matrix \( D \) appears in the place of \( K_p \), and an assumption can now be made about the signs of its entries \( d_1, d_2 \).

6.2 Controller Structure

Now, we formulate the adaptive controller parameterization for \( \dot{v}_f = [\dot{v}_f_1 \dot{v}_f_2]^T \). According to the SDU factorization approach \[9, 8\], a model matching control is expressed as

\[
\dot{\bar{v}}_f = \begin{bmatrix} \theta_1^T \Psi_1 \\ \theta_2^T \Psi_2 \end{bmatrix}, \quad \text{(44)}
\]

where \( \theta_1^* \in \mathbb{R}^3, \theta_2^* \in \mathbb{R}^2 \), \( \Psi_1 := [\omega_1^T \dot{\nu}_f] \) and \( \Psi_2 := \omega_f \). Hence, the (filtered) control parameterization is given by

\[
\dot{v}_f = \begin{bmatrix} \theta_1^T \Psi_1 \\ \theta_2^T \Psi_2 \end{bmatrix}, \quad \text{(45)}
\]
where \( \theta_i \) is the estimate of \( \theta^*_i \) with parameter error \( \hat{\theta}_i = \theta_i - \theta^*_i \) and the adaptation law is given by (i=1,2)

\[
\dot{\hat{\theta}}_i = -\gamma_i \text{sgn}(d_i) e_{ci} \Psi_i, \quad \gamma_i > 0,
\]

where \( \gamma_i \) are the adaptation gains and \( d_i \) are the entries of matrix \( D \). Now, in order to recover \( v \) in (35), we have

\[
\dot{v} = [\dot{v}_1 \ \dot{v}_2]^T \text{from (38) as}
\]

\[
\dot{v} = \left[ \bar{\theta}_1^T \Psi_1 + \bar{\theta}_2^T \Omega_1 \ \bar{\theta}_2^T \Psi_2 + \bar{\theta}_2^T \Omega_2 \right]^T,
\]

where \( \bar{\theta}_i \) is given by (46) and \( \Omega_i^T := [\omega^T \ \dot{v}_2]^T, \Omega_2 := \omega \). Note that, from (44) and (45), the state-error system (42)-(43) can be rewritten as:

\[
\dot{\bar{z}}_e = A_m \bar{z}_e + B_{m1} S \xi + B_{cs} \hat{J}(q) \sigma + B_{cb} w, \quad (48)
\]

\[
e_c = C_m \bar{z}_e, \quad (49)
\]

where \( \xi = \bar{\Theta}^T \Psi, \ \bar{\Theta}^T = [\bar{\theta}_1^T \ \bar{\theta}_2^T]^T \) and \( \Psi = \text{diag}(\Psi_1, \Psi_2) \). The corresponding algorithm is presented in Table 1.

### 6.3 Stability Analysis

Following [15], we consider the state representation (39)-(40). As a matter of fact, we can obtain a true Lyapunov function for the error system, with complete error state, by considering \( \omega_f = L^{-1}(s) \omega \). The error vector \( \dot{\omega}_f := L^{-1}(s)(\omega - \omega_M) \), where \( \omega_M \) corresponds to the model realization defined to achieve (36)-(37), can be expressed as the output of a stable and proper filter with input \( e_c \), and hence of \( \bar{z}_e \), similarly as in [15] p.358

\[
\dot{\varepsilon} = A_f \varepsilon + B_f \bar{z}_e, \quad \dot{\omega}_f = C_f \varepsilon + D_f \bar{z}_e,
\]

with \( \varepsilon \) of appropriate dimension and \( A_f \) being a strictly Hurwitz matrix. The passivity properties of the proposed adaptive visual servoing system is stated in the following theorem [8]:

**Theorem 3** Consider the systems (39)-(40) and (50), with adaptive control (35) and (47), and update law (46). Then, for \( P \) satisfying (41), the map \( B_{cs} \hat{J}(q) \sigma \mapsto \bar{P} \bar{z}_e \) is output strictly passive with positive definite storage function

\[
2V_L(\bar{z}_e, \varepsilon, \Theta) = \bar{z}_e^T P \bar{z}_e + \varepsilon^T P_1 \varepsilon + \Theta^T \Gamma^{-1} \Theta,
\]

where \( \Gamma = \text{diag}(\gamma_1, \gamma_2) \) and \( P_1 \) is a positive definite matrix satisfying

\[
A_f^T P_1 + P_1 A_f = -Q_1 \text{ for a positive definite matrix } Q_1.
\]

**Proof:** Considering \( V_L \) and for sufficiently small \( \alpha \) (using Schur’s complement [15]), the time-derivative of \( V_L \) is given by

\[
\dot{V}_L \leq -\bar{z}_e^T Q \bar{z}_e + \bar{z}_e^T P (B_{cs} \hat{J}(q) \sigma + B_{cb} w).
\]

Then, from the condition of persistent excitation of the kinematic regressor matrix \( W(q, \dot{q}) \), we have that \( b \to 0 \) and consequently \( w \to 0 \) and \( J \to J \). Now, the time-derivative of \( V_L \) assumes the form:

\[
\dot{V}_L \leq -\bar{z}_e^T Q \bar{z}_e + \bar{z}_e^T P B_{cs} \hat{J}(q) \sigma \quad \text{which, in turn, defines an output strictly passive map } B_{cs} \hat{J}(q) \sigma \mapsto \bar{P} \bar{z}_e.
\]

Now, considering also the passivity properties of the adaptive robot control by using Theorem 2, we can apply Theorem 1 where the cascaded subsystems \( \Sigma_1 \) and \( \Sigma_2 \) are identified by the corresponding states respectively as \( x_1 = [\bar{z}_e^T \ \dot{\varepsilon}^T \ \theta^T \ \theta^T]^T, y_1 = P \bar{z}_e \) and \( x_2 = [\dot{\theta}^T \ \dot{\theta}^T \ \eta]^T, y_2 = B_{cs} \hat{J}(q) \sigma \). Thus, from Theorem 1, all signals of the system are bounded and \( \sigma(t) \) and \( \bar{z}_e(t) \) tend to zero asymptotically. This implies that \( \lim_{t \to \infty} e(t) = 0 \) and \( \lim_{t \to \infty} e_c(t) = 0 \).

**Remark 1** In order to consider the Cartesian tracking problem in the image space, 3-DoFs have to be controlled by the visual servoing scheme. Thus, the image centroid could be used to provide planar tracking with respect to a desired image trajectory, whereas simultaneously the image area could be extracted to provide depth tracking with respect to a desired image depth [11].

### 7 Concluding Remarks

The problem of controlling robotic manipulators with non-negligible dynamics using adaptive visual servoing was presented. The proposed scheme was developed taking into account the uncertainties of both camera calibration and robot parameters. The kinematic control solution for the MIMO adaptive visual servoing case, without using image velocity information, is formulated as a relative degree two multivariable adaptive control problem.

The combination of the kinematic visual servoing scheme, based on the SDU factorization method, with the dynamics-based motion control for the robot was achieved by a cascade structure, resulting on an overall stable adaptive visual servoing system. The estimation of the uncertain Jacobian matrix was embedded in the proposed solution by using an indirect adaptive scheme. Simulation results are presented in order to illustrate the performance of the proposed control scheme in [http://www.coep.ufrj.br/~toni/results](http://www.coep.ufrj.br/~toni/results).

### References


Table 1
Algorithm for Adaptive Visual Servoing without image velocity measurement

<table>
<thead>
<tr>
<th>Regressor vector</th>
<th>$\omega = \frac{2\lambda \lambda_0^2}{\lambda_0^2} \psi c - (\lambda_0^2 + 2\lambda \lambda_0) \psi c + \lambda_0^2 r$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega^T_r = \omega^T$, $\Omega^T_i = [\omega^T \hat{v}_2]$.</td>
<td></td>
</tr>
<tr>
<td>Filtered signals</td>
<td>$\omega_f = L^{-1}(s) \omega$, $\hat{v}_{f1} = \theta^T_i \Psi_i$, $L(s) = (s + \lambda_i) I$.</td>
</tr>
<tr>
<td>Output error</td>
<td>$e_c = y_c - y_{cd}$, $y_{cd} = G_m(s) r$.</td>
</tr>
<tr>
<td>Robot Control law</td>
<td>$\tau = Y(q, \dot{q}, \ddot{q}, \dot{q}_r) \dot{a} - K_D \sigma$, $K_D = K_D^T &gt; 0$.</td>
</tr>
<tr>
<td>$e = q - q_m$, $\dot{q}_r = \dot{q}_m - \lambda e$, $\sigma = \dot{q} - \dot{q}_r = \ddot{e} + \lambda e$.</td>
<td></td>
</tr>
<tr>
<td>Cascade Strategy</td>
<td>$\dot{q}_r = J^{-1}(q) H^{-1}(s) [v + \lambda J(q) \dot{q}]$, $H(s) = (s + \lambda I) I$.</td>
</tr>
<tr>
<td>Visual Servoing law</td>
<td>$v_i = \dot{v}_i - 2\lambda_i \Lambda^{-1}(s) v_i$, $\dot{v}_i = \theta_i^T \Psi_i + \theta_i^T \Omega_i$, $\Lambda(s) = (s + \lambda_0) I$.</td>
</tr>
<tr>
<td>Adaptation laws</td>
<td>$\dot{\gamma}<em>i = -\gamma_i e</em>{cd} \Psi_i$, $\gamma_i &gt; 0$.</td>
</tr>
</tbody>
</table>


