OVERCOMING LIMITATIONS OF UNCALIBRATED ROBOTICS VISUAL SERVOING BY MEANS OF SLIDING MODE CONTROL AND SWITCHING MONITORING SCHEME
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ABSTRACT
This paper addresses the visual servoing control problem for robot manipulators without using velocity measurements, and considering a fixed but uncalibrated camera with an optical axis perpendicular to the robot workspace. A novel visual servoing strategy via sliding mode control (SMC) and a monitoring function based switching scheme is presented to deal with the uncertainties in the camera calibration parameters and to remove any restriction on the unknown camera orientation angle. The developed method provides global stability, disturbance rejection properties, and exact output tracking with better transient performance than adaptive controllers. Experimental results illustrate the robustness and practical feasibility of the proposed scheme.

Key Words: Visual servoing, robotic manipulators, uncalibrated camera, sliding mode control, switching adaptation.

I. INTRODUCTION
In recent decades, cameras have been raised as a useful sensor in order to locate and to inspect the objects of the environment without contact. Typically, in the robotics area, cameras are used for several applications in industry or in academic research. Following this trend, several tools for image processing have been developed and applied to the calibration, localization, measurement, and positioning. Thus, vision sensors can be used to increase the degree of autonomy and flexibility of robotic systems [1,2]. These abilities are necessary for robots to perform tasks in unstructured environments [3], as well as to share the workspace with human operators and other robots [4,5].

1.1 Previous works
For many years, the academic community and research groups have actively investigated the use of visual servoing applied to robotic arms, where the aim has been to control the pose of the end-effector with respect to a target object or a set of target features [6,7], or applied to mobile robots, where the goal has been to control the pose of the vehicle relative to some landmarks [8,9]. In this manner, the tracking problem of mobile targets or reference trajectories based on image features is an issue of interest, even for the planar case [10,11]. The feedback provided by vision sensors has been employed to develop several control strategies, using a fixed configuration (eye-to-hand) or a mobile configuration (eye-in-hand), even in the presence of uncertainties of the system parameters [12].

Several adaptive control schemes have been proposed to overcome the performance degradation due to modeling uncertainty, particularly with respect to the calibration parameters of the camera [13–19]. Nevertheless, in these adaptive methods, the camera misalignment/orientation angle must be restricted to the specific range \(-\frac{\pi}{2}, \frac{\pi}{2}\) or the restrictive (non generic) symmetry condition of the high-frequency gain matrix has to be circumvented by modifying known adaptive laws. Moreover, poor transient behavior and over-parameterization of the controller are limiting factors of adaptive schemes. Recently, some visual servoing strategies have been developed to deal with the over-parameterization problem based on the immersion and invariance technique [20,21] for adaptation laws, achieving global asymptotic tracking. Nevertheless, the proposed controllers are more involved when compared with the previous adaptive approaches and the other foregoing limitations remain unsolved.
As an alternative to cope with the uncertainties in both intrinsic/extrinsic camera parameters and to obtain fast convergent responses, the sliding mode control (SMC) approach [22,23] can be used to obtain robust visual servoing schemes [24]. Since this control methodology utilizes signal domination techniques or signal synthesis instead of the gradient based control law normally used in adaptive schemes [25], the former can guarantee good transient performance, along with providing robustness properties in the presence of unmodeled dynamics, input disturbances, and parametric uncertainty. The intrinsic difficulties of classical SMC, such as control chattering, could be alleviated by higher order sliding mode (HOSM) control [26] or boundary layer methods [22]. It is known that ideal sliding modes are not realizable in practice due to physical imperfections, such as delays, noise, unmodeled dynamics, and discretization. Nevertheless, several methods allow chattering alleviation and these difficulties do not preclude applicability of SMC to real systems, as has been frequently shown in the literature [22,23].

In particular, widening the limits for the camera misalignment angle seems to be of interest when the task has to be performed in unstructured environments, where the physical or geometrical description of the workspace is not known fully a priori. For structured environments, a number of visual servoing strategies developed in the case of parallel planar robot manipulators assume knowledge of the matrix that describes the rotation between the camera and the robot planes [27–29]. On the other hand, a visual servoing scheme based on SMC was developed in [30] and the aforementioned drawback could be circumvented in an ad hoc manner, i.e., a formal justification of the controller for uncertain camera misalignment was not presented. In addition, only local convergence properties could be guaranteed and the control design required the measurement of the image velocity, which may not be easy in practice due to noisy image data and increased delay in the control loop [29,31].

Most of the works in robot visual servoing have focused on kinematic control [32]. Exceptions include dynamic image-based visual servoing of robot manipulators when the camera parameters as well as robot physical parameters are uncertain. In addition, more general problems can be found, e.g., when uncertainties are presented in the constraint surface, kinematics, dynamics, and camera model [33,34]. In the present paper, for simplicity, we reduce the problem to kinematic control, an assumption that is justified when applied to industrial robots, which have high-gear ratios or when the task speed is not too fast [35]. If this is not the case, the performance might deteriorate in view of the unmodeled robot dynamics. Robot dynamics, however, could be included, as in [17,19,21].

1.2 Contribution

In this work, the visual servoing control problem for robot manipulators using an uncalibrated camera that is fixed to the ground and possessing an optical axis perpendicular to the robot workspace is considered.

A novel visual servoing control strategy based on sliding modes and a switching monitoring scheme is presented to deal with the uncertainties in the camera calibration parameters and to remove the classic restriction for the existing adaptive visual servoing schemes, which require the unknown camera misalignment angle to belong to some strict interval [15–19].

As in supervisory control [36], the proposed switching mechanism selects a suitable static pre-compensator out of a finite indexed set of matrices, according to an appropriate monitoring function, to correct any mismatch from the camera nominal orientation. Also, different from previous adaptive based controllers, the proposed switching adaptation control law is robust to a general class of input disturbances, including perturbations with unknown bounds.

The presented scheme is developed for image-based look-and-move visual servoing systems. Global stability properties and exact tracking can be guaranteed. In particular, the transient performance of the tracking error can be significantly improved by tuning a design parameter of the monitoring function. Experimental results are shown to illustrate the practical performance and feasibility of the proposed control scheme in a real-world scenario.

1.3 Notation and terminology

The following notation and basic concepts are employed throughout the paper.

- Class-$\mathcal{K}$ functions are defined as in [37, p. 144].
- The Euclidean norm of a vector $\mathbf{x}$ and the corresponding induced norm of a matrix $\mathbf{A}$ are denoted by $|\mathbf{x}|$ and $|\mathbf{A}|$.
- We adopt Filippov’s definition for the solution of discontinuous differential equations [38] and the concept of extended equivalent control [39], which is also applicable to the reaching phase of a sliding mode.
- $\mathbf{E}_a = [\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a]$ denotes the orthonormal frame $a$ and $\mathbf{x}_a, \mathbf{y}_a, \mathbf{z}_a$ denote the unit vectors of the frame axes.
- For a given vector $\mathbf{v} \in \mathbb{R}^n$, its elements are denoted by $v_i$ for $i = 1, \ldots, n$, that is, $\mathbf{v} = [v_1, v_2, \ldots, v_n]^T$.

II. VISUAL SERVOING KINEMATIC CONTROL: A REVIEW

First, consider the kinematic control problem for a robot manipulator. In this framework, the robot motion can be described by [35]:

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\[
\dot{\theta}_i = v_i, \quad i = 1, \ldots, n, \tag{1}
\]
where \( \theta_i \) and \( \dot{\theta}_i \) are the angular position and the angular velocity of the \( i \)th joint, respectively, and \( v_i \) is the velocity control signal applied to the \( i \)th joint motor drive.

Remark 1 (Approximate kinematic model). Note that (1) is an approximate dynamic model for control design purposes. Thus, even if the voltage applied to the motor is discontinuous, this does not mean that the true velocity is discontinuous or that the torque applied to the manipulator is infinite. For a clear discussion about this model and the neglected dynamics in the SMC context, see [23, chapter 8]. Generalization to consider the robot dynamics could be made as in [17,19,21].

Now, let \( x \in \mathbb{R}^m \) be the end-effector position with respect to the robot base, expressed in the base frame \( \mathcal{E}_b \) given by the forward kinematics map \( x = f(\theta) \), where \( \theta \in \mathbb{R}^n \) is the vector of manipulator joint angles. In general, \( f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is a nonlinear transformation describing the relation between the joint space \( \mathcal{J} \) and the operational space \( \mathcal{O} \) (or Cartesian space). Note that the dimension of the joint space \( \mathcal{J} \) is \( n \) whereas the dimension of the operational space \( \mathcal{O} \) is \( m \), that is, \( \dim(\mathcal{J}) = n \) and \( \dim(\mathcal{O}) = m \).

The differential kinematics equation can be obtained as the time derivative of the forward kinematics given by
\[
\dot{x} = J(\theta)\dot{\theta} = J(\theta)v, \quad J(\theta) = \frac{\partial f(\theta)}{\partial \theta} , \tag{2}
\]
where \( J(\theta) \in \mathbb{R}^{m \times n} \) is the manipulator Jacobian.

A Cartesian control law \( u(t) \) can be transformed to joint control signals via
\[
v = J^{-1}(\theta)u, \tag{3}
\]
and \( u(t) \) can be designed to control the end-effector position. Here, the following two assumptions are considered:

A1. the robot kinematics is known;

A2. the control law \( u \) does not drive the robot to singular configurations [35].

In this work, the visual servoing approach is used to provide closed-loop position control for the robot end-effector with an eye-to-hand camera configuration. Fig. 1 illustrates the camera, image, and robot frames, as well as the camera calibration parameters, such as the camera orientation angle \( \psi \) (or camera misalignment) with respect to the robot base frame \( \mathcal{E}_b \), the focal length of the camera lens \( f_o \), and the constant depth \( z_0 \) between the image frame \( \mathcal{E}_i \) and the robot working plane (in general \( z_0 \gg f_o \)). Here, we assume that the camera frame \( \mathcal{E}_c \) and the robot base frame \( \mathcal{E}_b \) are aligned only with respect to the \( z \)-axis.

Let \( y \in \mathbb{R}^2 \) be the position of the image feature fixed on the robot end-effector expressed in the image frame \( \mathcal{E}_i \), and let \( l \) be the dimension of the image space \( \mathcal{I} \). Here, we will consider that the robot structure performs only planar motions constrained to the working plane and it is monitored by a single fixed camera. Hence, without loss of generality, we have \( m = n = l = 2 \), which implies \( \theta \in \mathbb{R}^2 \), \( x \in \mathbb{R}^2 \), \( y \in \mathbb{R}^2 \), and \( J(\theta) \in \mathbb{R}^{2 \times 2} \), respectively. Note that, in this case, the robot structure is non-redundant [35] and \( \dim(\mathcal{J}) = \dim(\mathcal{O}) = \dim(\mathcal{I}) = 2 \).

From the monocular reconstruction of a pinhole type camera with optical axis perpendicular to the robot workspace and assuming that all lens distortions and aberrations are negligible [10], the operational space \( \mathcal{O} \) can be related to the image space \( \mathcal{I} \) by [4]
\[
y = K_p x + y_0 , \tag{4}
\]
where \( K_p \) is the so-called camera workspace transformation matrix given by
\[
K_p = \begin{bmatrix} h_1 \cos(\psi) & -h_1 \sin(\psi) \\ h_2 \sin(\psi) & h_2 \cos(\psi) \end{bmatrix} , \tag{5}
\]
with
\[
h_i = \alpha_i \frac{f_o}{z_c}, \quad z_c = f_0 + z_0, \quad i = 1, 2
\]
where \( \alpha_i \), \( \alpha_1 > 0 \) are the camera scaling factors (in pixel per millimeter) and \( y_0 \) is a constant term, which depends on the
position of the camera frame $E_c$ with respect to the robot base frame $E_b$.

Now, consider the robotics visual servoing control problem with a fixed and uncalibrated camera mounted in the robot workspace. Here, we assume that the control goal is to follow a desired time-varying trajectory $y_m(t) \in \mathbb{R}^2$ from the current position of the robot end-effector $y$, both expressed in the image frame $E_c$. The control goal can be described simply by

$$y \rightarrow y_m(t), \quad e = y - y_m \rightarrow 0,$$

(6)

where $e \in \mathbb{R}^2$ is the image error. Considering that $\psi$ and $z_c$ are constant, the above transformation (4) is linear and time-invariant. From (2) and (3), the Cartesian control problem in the image space $I$ can be described by

$$\dot{y} = K_p\mu,$$

(7)

where $u \in \mathbb{R}^2$ is the control signal to be designed.

Thus, using a feed-forward plus proportional control law

$$u = K_p^{-1}[\dot{y}_m + K(y_m - y)],$$

(8)

the image error dynamics is given by $\dot{e} + Ke = 0$. Hence, for a positive definite matrix $K$, we have $e(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$. Nevertheless, if the intrinsic and extrinsic parameters of the camera are uncertain (uncalibrated camera case), the camera-workspace transformation matrix $K_p$ is also uncertain. Therefore, a linearizing control law cannot be implemented directly, as in (8); thus, the asymptotic convergence of the image error is not guaranteed.

In this context, some adaptive schemes have been proposed in order to cope with the uncertainty in the camera calibration parameters [7–21]. Nevertheless, it is known that the adaptive strategies can lead to bad transient behavior in addition to the lack of robustness with respect to unmodeled dynamics and input disturbances. Moreover, in most of the existing adaptive visual servoing methods, the camera orientation angle $\psi$ must be restricted to the range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

In what follows, the combination of the proposed SMC approach and the switching monitoring scheme is applied to circumvent the usual limitations of adaptive control methods, particularly with respect to bad transient behavior, lack of robustness, and restricted range of camera misalignment angle.

### III. PROBLEM FORMULATION

This paper considers the global tracking problem of the multiple input multiple output (MIMO) nonlinear system given by

$$\dot{y} = K_p\mu + \phi(y, t),$$

(9)

where $u \in \mathbb{R}^2$ is the control input, $y \in \mathbb{R}^2$ is the measured output, and $\phi$ is an uncertain nonlinear disturbance, piecewise continuous in $t$ and locally Lipschitz continuous in $y$, that was included in (7) to show the disturbance rejection capability of the proposed controller. More general systems than (9) could be considered, as done in [40].

For each solution of (9), there exists a maximal time interval of definition given by $[0, t_M)$, where $t_M$ may be finite or infinite. Thus, finite-time escape is not precluded a priori.

#### 3.1 Basic assumptions

Besides the usual assumptions, A1 and A2, it is further considered that the constant matrix $K_p$ is unknown (we also say that the plant has unknown control direction) in the sense that all uncertain parameters of the matrix $K_p$ belong to some compact set $\Omega_p$. In $\Omega_p$, it is only assumed that $A3$. (i) $\det(K_p) \neq 0$, (ii) there exists a known constant $c > 0$ such that $|K_p| \leq c$, and (iii) there exists a finite indexed set $Q$ of known matrices $S_q \in \mathbb{R}^{2 \times 2}$ such that $-K_pS_q$ is Hurwitz for some $q \in Q$.

The Hurwitz condition in Assumption A3 is necessary and sufficient for the attractiveness of the sliding surface in the case of unit vector sliding mode control [39,41,42]. This assumption significantly relaxes the usual requirement of positive definiteness and symmetry of some common adaptive control algorithms [43,44]. Symmetry is a non-generic property, and it can be destroyed easily by arbitrarily small uncertainties in $K_p$. Moreover, if $K_p$ is positive definite, then this implies that $-K_p$ is Hurwitz, but the converse is not true. The existence of the finite indexed set $Q$ is guaranteed by the spectrum-unmixing sets results presented in [45,46].

In the particular case of uncalibrated visual servoing, $K_p$ satisfying A3 results in an arbitrary and unknown/uncertain camera misalignment angle. In such a case, a switching mechanism based on the monitoring function will be provided for cycling through the elements of the finite indexed set $Q$ so that the control objectives are achieved.

In order to obtain a norm bound for $\phi$ in (9) we additionally assume that

#### A4. There exist a known class-$\mathcal{K}$ function $\varphi(\cdot)$ and an unknown constant $c_0 \geq 0$ such that $|\varphi(y)| \leq \varphi(|y|) + c_0 + |\pi(t)|$, where the signal $\pi(t)$ is also unknown and exponentially decreasing, i.e., $|\pi(t)| \leq R e^{-\lambda t}$, $\forall t$, for some positive (unknown) scalars $R$ and $\lambda$.

Note that A4 is not restrictive since $\phi$ is assumed to be locally Lipschitz continuous in $y$. The domination or bounding
function $\varphi$ does not impose any particular growth condition w.r.t. the system output. Thus, polynomial nonlinearities in $y$ can also be coped with. Furthermore, the system input disturbance may contain vanishing, as well as uniformly bounded terms, but otherwise is unknown (with unknown bounds).

The motivation to include the vanishing term is to account for possible transient signals present in the disturbance or due to the initial conditions of the plant internal states. The latter might appear if plants of higher order were to be considered [40].

### 3.2 Global tracking problem

The problem consists of designing a control law $u$, without the knowledge of the plant control direction, to drive the output tracking error

$$e(t) = y(t) - y_m(t)$$

asymptotically to zero (exact tracking), starting from any plant or controller initial conditions and maintaining uniform closed-loop signal boundedness, in spite of uncertainties and input disturbances.

The desired time-varying trajectory $y_m(t)$ is assumed to be generated by the reference model:

$$\dot{y}_m = A_m y_m + r, \quad A_m = -\text{diag}(\gamma_1, \gamma_2),$$

where $\gamma_1, \gamma_2 > 0$ and $r(t) \in \mathbb{R}^2$ is assumed piecewise continuous and uniformly bounded.

### 3.3 From tracking to regulation

From (9)–(11) , the $e$-dynamics can be written as:

$$\dot{e} = A_m e + K_p (u - u^*),$$

where

$$u^* := K_p^{-1}(-\phi + A_m y + r).$$

Then, the global tracking problem can be reformulated as the regulation problem described in the following. Find a sliding mode control law $u$ in such a way that, for all initial conditions: (i) the solutions of (9) and (12) are bounded; and (ii) $e(t)$ tends asymptotically to zero.

The ideal control $u^*$ is considered as a matched input disturbance in (12). From A3–A4, it can be norm bounded by

$$|u^*| \leq c [\varphi(y)] + c_p |\pi| + |A_m y + r|,$$

where $c$ is defined in A3 and $\varphi$, $c_p$, $\pi$ in A4.

### IV. VISUAL SERVOING SLIDING MODE CONTROL

Since $\psi$ and $K_p$ are both unknown, let $q^*$ be the unknown index of the indexed set $Q$, given in A3, for which the corresponding unknown matrix $S^* = S_{q^*}$ assures that $-K_p S^*$ is Hurwitz. Thus, the Lyapunov equation $(K_p S^*)^TP + P(K_p S^*) = I$ has a solution $P = P^T > 0$. Now, if the control direction is known ($\psi$ and $q^*$ are known) we can apply the unit vector control (UVC) law [39,41]

$$u = -S\varphi(y, t) \frac{e}{|e|}, \quad \forall t \in [0, t_M),$$

and (12) and verify that, if the modulation function $\rho$ satisfies

$$\rho \geq c_p |u^*| + c_p |\pi| + \delta - c_d |\pi|, \quad \delta \geq 0,$$

where $c_d$, $c_p$ and $\delta$ are constant terms and $c_d c |\pi|$ is an exponential decaying term, then the time Dini derivative [37] of $V = \sqrt{e^T Pe}$ along the solutions of (12) satisfies,

$$\dot{V} \leq -\lambda_m V + \frac{c_d |\pi|}{2\sqrt{\lambda_{\text{max}}(P)}}, \quad \forall t \in [t, t_M),$$

for any $t \in [0, t_M)$, where $\lambda_m > 0$ is any arbitrary constant,

$$\lambda_{\text{max}}(P) (\lambda_{\text{max}}(P))$$

denotes the minimum (maximum) eigenvalue of $P = P^T > 0$, and $\pi$ comes from (14). Hence, using the comparison lemma [38], we have:

$$|e(t)| \leq \zeta(t),$$

$$\zeta(t) := c_p |e(t)| e^{-\lambda_m(t-t^*)} + |\pi(t)|, \quad \forall t \in [t, t_M),$$

where $c_p > \sqrt{\lambda_{\text{max}}(P)/(\lambda_{\text{max}}(P))}, \pi(t)$ is also an exponentially decreasing term, i.e., $|\pi(t)| \leq R e^{-\lambda_M t}, \forall t$, for some positive scalars $R$ and $0 < \lambda_M \leq \min\{\lambda, \lambda_m\}$, with $\lambda$ unknown given in A4. For more details, see Appendix A.

Note that the major difficulty is that $q^*$ is unknown; thus, the UVC law (15) cannot be implemented. In [47] and [48], a switching scheme based on the monitoring function was developed to cope with the lack of knowledge of the control direction, where only linear SISO plants were considered.

In that case, $K_p$ was a scalar, and after a finite number of changes in the control sign ($S_\psi = \pm 1$), the correct control direction could be detected. For MIMO nonlinear plants, the UVC law is redefined as:

$$u = -S\varphi(y, t) \frac{e}{|e|}, \quad \forall t \in [0, t_M),$$

where $\rho$ satisfies (16) and a switching mechanism also based on a monitoring function is used to decide when the static...
pre-compensator matrix $S_q$ [36] should be switched within the collection of matrices with $q \in Q$ [45,46].

**Remark 2 (Switching adaptation).** Notice that, since $u$ in (20) is a switching adaptation based control law, the proposed visual servoing scheme via sliding modes can be advantageous when compared to well-known integral adaptive control laws. In general, the traditional adaptive control schemes are subject to over-parameterization or depend on complex adaptation laws, appropriate factorization of the high-frequency gain matrix, and persistent excitation conditions over the regressor vector.

**V. SWITCHING SCHEME AND MONITORING FUNCTION**

Now, we can construct the monitoring function $\varphi_m$ based on the norm bound for $e$ given in (19) following the ideas developed in [47,48]. Remembering that (19) holds when the matrix $S_q$ is correct ($S_q = S$), it seems natural to use $\xi$ as a benchmark to decide whether a switching of $S_q$ is needed, that is, the switching occurs only when (19) is violated.

Nevertheless, since $S_q$ is unknown and $\pi$ is not available for measurement, we consider the following function, defined in the interval $[t_0, t_M)$, to replace $\xi$:

$$\varphi(t) = c_p |e(t)| e^{-\omega t} + a(k) e^{-\frac{t}{\pi(k)}},$$

where the switching time $t_s$ sets the change of index $q \in Q$, thus cycling through the $S_q$ matrices, and $a(k)$ is any positive monotonically increasing unbounded sequence.

The monitoring function $\varphi_m$ can be defined as:

$$\varphi_m(t) = \varphi(t), \quad \forall t \in [t_0, t_{M+1}) \subseteq [0, t_M).$$

Note that, from (21) and (22), we have $|e(t_s)| < \varphi(t_s)$ at $t = t_s$. Hence, the switching time $t_s$ is defined as the time instant when the monitoring function $\varphi_m(t)$ meets $|e(t)|$, that is,

$$t_{s+1} := \min \{ t > t_s : |e(t)| = \varphi_m(t) \}, \quad \text{if it exists},$$

$$t_M, \quad \text{otherwise},$$

where $k \in \{0, 1, \ldots\}$ and $t_0 := 0$ (see Fig. 2). The following inequality is directly obtained from (22):

$$|e(t)| \leq \varphi_m(t), \quad \forall t \in [0, t_M).$$

Fig. 2 illustrates the tracking error norm $|e|$, as well as the monitoring function $\varphi_m$.

**Remark 3 (Controller design parameters).** The monitoring function $\varphi_m$ (21)-(22) is used to switch the matrix $S_q$ in (20). The minimum finite set of matrices $S_q$, $q \in Q = \{0, 1, 2, 3\}$ that can be chosen in our control problem is

$$S_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, S_3 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

For any arbitrary camera misalignment $\psi$, $-K_p S_q$ is Hurwitz for some $S_q$; therefore, the usual restriction $|\psi| < \frac{\pi}{2}$ rad can be removed. Using (14), one alternative modulation function satisfying (16) is

$$\varphi = c_p |y| + a(k) + |A_w y + r| + c_r |y| + \delta,$$

which can be applied to the implementation of the proposed controller, where $c_p$ and $c_r$ are defined according to (18) and $\delta \geq 0$ is any arbitrary small constant. Notice that the use of the term $a(k)$ in (25) is motivated by the fact that, after some finite number of switchings, $a(k)$ would become an upper bound for the unknown constant bound $c_p$ for the disturbance assumed in $A_4$.

Since $u$ in (20) and the monitoring scheme do not depend on $\dot{y}$ or $\dot{e}$, the visual servoing controller can be designed free of image velocity measurement (or optical flow) of the robot manipulator, which may be advantageous in practice.

Fig. 3 presents the block diagram of the proposed vision-based control system in the presence of uncertainties in the camera model.

**Remark 4 (Unboundedness observability property).** The closed-loop system has an unboundedness observability property in the sense that any infinite or finite-time system signal escape can be observed in the system output $y(t)$. Thus, any eventual escape is precluded if $y(t)$ (or the output tracking error $e(t)$) remains uniformly bounded. Hence, by the “exponentially decaying feature” of the monitoring function (21)-(22), it is clear that unbounded solutions for $e(t)$ can only occur if $\varphi_m(t)$ performs an infinite number of switchings.

![Fig. 2. The trajectories of $\varphi_m$ and $|e|$](image-url)
(k \rightarrow + \infty). In addition, if the switching process stops, then e(t) \rightarrow 0 exponentially, as will be shown in the next section.

VI. STABILITY ANALYSIS

The main result is now stated by the following theorem.

**Theorem 1.** Assume that A1–A4 hold. Consider the plant (9) with UVC law (20) and monitoring function (21)–(22). If the modulation function (25) is applied, then the control direction switching stops, assuring that all signals in the closed loop system remain uniformly bounded and the tracking error e(t) is globally exponentially convergent to zero. Moreover, if \( \delta > 0 \) in (25), then the ideal sliding mode on the manifold e = 0 is reached in finite time.

**Proof.** The proof is carried out in three parts.

**I. The monitoring function switching stops.** Motivated by Remark 4, suppose by contradiction that \( S_q \) in (20) switches without stopping \( \forall t \in [0, t_M] \), where \( t_M \) may be finite or infinite. Then, \( a(k) \) in (21)–(22) and (25) increases unboundedly as \( k \rightarrow + \infty \). Thus, there is a finite value \( \kappa \) such that, for \( k \geq \kappa \): (i) \( a(k) e^{-\delta k} \geq |\epsilon_r(t)| \) in (19); (ii) \( a(k) \geq c_0 \) in (14), (16) and (25); (iii) \(-K_p S_q \) is Hurwitz. In this case, from (i), \( \varphi_m(t) \geq \zeta(t) \), \( \forall t \in [t_{\text{init}}, t_{\text{end}}] \), with \( \zeta \) in (19). Moreover, from (ii) and (iii), \( \zeta \) is a valid upper bound for \(|\epsilon|\). Hence, no switching will occur after \( t = t_{\text{ns}} \), i.e., \( t_{\text{end}} = t_M \) [see (23)], which leads to a contradiction. Therefore, \( \varphi_m \) to (22) has to stop switching after some finite \( k = N \) and \( t_N \in [0, t_M] \).

**II. Closed loop signal boundedness and ultimate exponential convergence to zero.** Since the switching stops and \( \varphi_m \) converges to zero exponentially, we conclude (independently of whether a Hurwitz matrix \(-K_p S_q \) is selected at \( k = N \) or not) that e(t) will converge to zero at least exponentially. Remembering that \( y = e + y_m \) and \( y_m \) is uniformly bounded, we can conclude that y, u, and all closed-loop system signals are also globally uniformly bounded and cannot escape in finite-time, i.e., \( t_{\text{ns}} \rightarrow + \infty \).

**III. Ultimate \( S_q \) selection and ideal sliding mode.** We know that, if \(-K_p S_q \) is Hurwitz, all trajectories of the system converge to the origin of the error state space [41, lemma 1]. Moreover, if \(-K_p S_q \) is not Hurwitz, then, for almost every initial condition (i.e., except for a set of zero measure), the system trajectories diverge unboundedly or do not converge to the origin. This is a contradiction, since, if the switching stops, according to Part II of the proof, the state e(t) must converge to the origin. Then, almost always, the ultimate matrix \( S_q \) selected is such that \(-K_p S_q \) is Hurwitz. Thus, also from [41, lemma 1], we can conclude further that e becomes zero after a finite time, provided that \( \delta > 0 \) in (25), satisfying (16). To reduce the sliding mode reaching time, \( \delta \) should be increased. The trade-off is the control effort, which would also increase.

**Remark 5 (Expected transient performance).** Notice that, due to the initial identification phase of a stabilizing \( S_q \), it is not possible to state Lyapunov stability with respect to the state space origin. Indeed, even if the initial state norm is small, we cannot guarantee that the initial transient is correspondingly small during the initial phase. Nevertheless, either the trajectories start in an invariant compact set or they tend to this set asymptotically. Ultimately, the trajectories converge exponentially to the origin, as stated in Theorem 1.

Furthermore, if the input disturbance \( \phi \) assumed in A4 had completely known upper bounds, the term \( a(k) \) used in (25) would be replaced by the known upper bound available \( c_0 \) and the exponential \( a(k) e^{-\delta k} \) applied to the monitoring function (21) could be removed. In this case, any transient performance pre-specifications for the tracking error could be guaranteed. Thus, different from standard adaptive control systems, the proposed switching adaptation based controller could provide arbitrarily good transient and steady-state responses.

**Remark 6 (Measurement noise issues).** The monitoring function \( \varphi_m \) (21)–(22) must be redesigned in order to avoid the occurrence of spurious switchings due to the presence of measurement noise, which was originally neglected in the theory. Since the monitoring function is based on an upper bound for the output tracking error, it seems natural to include some rough norm bound of the noise signal to obtain a monitoring function coping with it. By adding a constant \( c_n > 0 \) to the monitoring function (21), i.e.,
\[ \varphi_k(t) = c_k |e(t)| e^{-\lambda_n(t-t)} + a(k) e^{-\frac{1}{\alpha(t)}} + c_m \]

only practical tracking can be assured with respect to a residual set of order \( \mathcal{O}(c_m) \) if the monitoring function switching stops. Nevertheless, such a redesigned scheme becomes more efficient since it avoids the switchings having to be restarted after the correct gain matrix has already been identified and practical convergence obtained. Consequently, the switching process lasts at most one cycle less throughout the indexed set \( Q \).

VII. EXPERIMENTAL RESULTS

This section describes the experimental setup and discusses the obtained test results, which illustrate the performance of the proposed control scheme in a real scenario, in the presence of parametric uncertainties, unmodeled robot dynamics, and nonlinearities (e.g., backlash and friction). As a remarkable feature, we will show that an arbitrary camera misalignment angle is allowed during the experiments.

7.1 Experimental setup

The proposed controller was implemented on a 6-DoF (six Degrees of Freedom) Zebra Zero robot manipulator (Integrated Motions, Inc., Berkeley, CA, USA) performing planar motions on a vertical plane. Thus, the robot base and the end-effector orientation were kept constant during the experiment and only the shoulder and elbow joints were rotated. Due to the robot’s large gear ratios and a high gain velocity control loop [49], the effects of robot dynamics were considered negligible in the control design; thus, a purely kinematic approach was adopted. The robot task involved the visual tracking of a reference trajectory from a target feature fixed on the robot wrist. A KP-D50 CCD camera (Hitachi, Ltd., Tokyo, Japan) with a lens of focal length \( f_0 = 6 \text{ mm} \) and scaling factors \( \alpha_1 = 119 \text{ pixel mm}^{-1} \) and \( \alpha_2 = 102 \text{ pixel mm}^{-1} \) was mounted on the floor in front of the Zebra Zero. Fig. 4 shows the camera point-of-view corresponding approximately to the orientation \( \psi \approx 0 \text{ rad} \).

The average depth from the image plane to the vertical plane in the robot workspace was \( z_0 = 1 \text{ m} \). The extracted visual features are the centroid coordinates of the image of a red sphere fixed on the robot wrist. The images with \( 640 \times 480 \text{ pixel} \) were acquired using a Meteor frame-grabber (Matrox, Ltd., Dorval, Quebec, Canada) at 30 frames per second (FPS).

The visual servo controller was coded in C language and executed in 35 ms on a 200 MHz Pentium Pro processor with 64 Mbyte RAM. The joint velocity command generated by the visual servoing control law fed the Zebra Zero ISA board, which closed a velocity loop using an HCTL1100 micro-controller (HP Inc.) working in proportional velocity mode with 0.52 ms sampling time.

The image processing in RGB format was performed on a sub-window of \( 100 \times 100 \text{ pixel} \). The first estimation of the centroid coordinates was performed offline using an ad hoc Graphical User Interface developed in Tcl/Tk language [50], as shown in Fig. 4. During the task execution, the feature was computed using the image moments algorithm [51]. Due to noise sensitivity, the proportional gain in the velocity loop could not be made high enough to eliminate the steady state error [49] due to gravity effects acting in joints 2 and 3 (shoulder and elbow). The gravity disturbance was identified offline using a least squares method and was effectively canceled [52].

7.2 Test results

The experimental tests were performed without previous calibration of the camera parameters. The desired trajectory \( y_m \) was a circle generated by the model (11), with \( \gamma_1 = \gamma_2 = 1 \) and \( r^T = [r_1 \ r_2] \) with

\[
\begin{align*}
    r_1 &= y_1(0) + R[1 - \cos(\omega_t t)], \\
    r_2 &= y_2(0) + R[\sin(\omega_t t)],
\end{align*}
\]

where \( y^T(0) = [y_1(0) \ y_2(0)] \) is the initial position of the centroid coordinates in the image frame, and where \( R \) and \( \omega_t \) determine the radius and the angular velocity of the reference trajectory, respectively. The robot manipulator should then perform the tracking of a circular trajectory specified by \( R = 40 \text{ pixel} \) and \( \omega_t = \frac{\pi}{5} \text{ rad s}^{-1} \). The initial position of the centroid was \( y_1(0) = 330 \text{ pixel} \), \( y_2(0) = 275 \text{ pixel} \) and the camera orientation intentionally was changed to different values by software, without modifying the controller, in order to verify the effectiveness of the adaptation scheme.
The monitoring function $\phi_m$ was obtained from (21)–(22) with $a(k) = k + 1$ and $c_p = 2$. In addition, a constant $c_n = 15$ pixel was added to $\phi_m$ to reduce spurious modifications in the control direction estimate due to the measurement noise. It is known that the measurement noise and the low sampling rate of the CCD camera can cause control chattering, which can be alleviated by use of a boundary layer in the UVC law [22,23], resulting in a continuous control signal with finite torques. The modulation function $\rho$ in (20) was implemented in order to satisfy (25) and to deal with possible unknown disturbances (see A4) present in the experiments. Since no output-dependent disturbance was considered, $\phi(|y|)$ was set to zero. All tests were done to avoid the manipulator Jacobian singularities in (3).

Fig. 5 shows the time history of the monitoring function $\phi_m$ and the error norm $|e|$. The experimental test was performed with $\psi = \pi$ rad, while the nominal value was assumed to be $\psi = \pi/2$ rad. Thus, we had a very large mismatch from the “nominal” orientation. For the nominal orientation, the correct matrix pre-compensator would be $S_0$, so this was the initial pre-compensator applied to the controller. Therefore, it would be necessary to accomplish three switchings to reach the correct matrix $S_1$ (for $\psi = \pi$ rad) and guarantee stable model following. Nevertheless, in order to test the cyclic switching and the robustness of the proposed scheme under a time-varying control direction, the camera misalignment angle was set to $\psi = \pi/2$ rad in the last portion of the experiment, before the third switching. Note that, at the fourth switching (4th SW), the correct matrix $S_0$ (for $\psi = \pi/2$ rad) was selected again ($-K_pS_0$ is Hurwitz) and $|e|$ vanishes thereafter.

Fig. 6 describes the time history of the image error $e$ and the control signal $u$, respectively. Note that, in spite of the transient behavior, the asymptotic convergence of error to a small residual set about 5 pixel was achieved after $t = 60$ s. The residual set was relatively small and perfectly compatible with the practical performance expected from a SMC under a realistic scenario where measurement noise, numerical discretization, and boundary layer relay approximation are present. The trajectory tracking is illustrated in Fig. 7, where it is possible to verify that, after the fourth switching, a remarkable performance was achieved.

**VIII. CONCLUSION**

A new sliding mode control strategy based on a switching scheme and monitoring function was developed to tackle
the uncalibrated visual servoing problem for robot manipulators. In particular, the usual restriction for the existing adaptive visual servoing approaches, which require the uncertain camera orientation angle being bounded away from the critical values $\pm \pi/2$ rad, can be removed in the new scheme. The proposed vision-based control system leads to global stability, finite-time convergence of the tracking error state to zero with transient-performance improvement, and strong robustness to large calibration uncertainty and input disturbances. The theoretical results are corroborated by successful experimental testbeds with a robotics visual servoing setup without any kind of pre-calibration.

Future research topics following the ideas developed here are: (a) to relax the complete knowledge of the robot kinematics; (b) to include the nonlinear uncertain robot dynamics in the control design; (c) to apply the proposed switching monitoring scheme to the eye-in-hand camera configuration; and (d) to consider the optical axis of the camera non-perpendicular with respect to the robot workspace resulting in a state-dependent camera-workspace transformation matrix.

IX. APPENDIX A

9.1 Proof of Inequalities (17) and (19)

If $-K_p S$ is Hurwitz, there exists $P = P^T > 0$ such that $(K_p S)^T P + P (K_p S) = I$. Thus, consider the quadratic form $W(e) = e^T P e$, which has its time derivative along the solutions of (12) given by

$$\dot{W} = -\dot{\varphi} |e| + e^T (A_{e}^r P + P A_{e}) e - 2e^T P K_p u^*.$$  \hspace{1cm} (28)

Moreover, $\dot{W}$ satisfies the following inequality

$$\dot{W} \leq -\varphi |e|^2 + |A_{e}^r P + P A_{e}| |e|^2 + 2|e| |P K_p u^*|.$$  \hspace{1cm} (29)

If the modulation function $\varphi$ satisfies (16), i.e.,

$$\varphi \geq c_e |u^*| + c_i |e| + \delta - c_d e |\pi|, \quad \delta \geq 0,$$

then, we can write:

$$\dot{W} \leq (|A_{e}^r P + P A_{e}| - c_d) |e|^2 + 2|e| |P K_p u^*|$$

$$\quad - \delta |e| + c_i e |\pi| |e|.$$  \hspace{1cm} (31)

Furthermore, choosing the constants $c_e, c_i, c_d$ according to (18) we obtain:

$$\dot{W} \leq -2\lambda_{\text{max}} (P) \lambda_{\text{min}} |e|^2 + c_i e |\pi| |e|.$$  \hspace{1cm} (30)

Now, in view of the Rayleigh-Ritz inequality

$$\lambda_{\text{min}} (P) |e|^2 \leq W(e) \leq \lambda_{\text{max}} (P) |e|^2,$$

inequality (29) can be rewritten as:

$$\dot{W} \leq -2\lambda_{\text{min}} (P) \lambda_{\text{min}} |e|^2 + c_i e |\pi| |e|.$$  \hspace{1cm} (31)

Then, defining $V := \sqrt{W}$, the time Dini derivative $\dot{V}$ satisfies (17). Hence, using the comparison lemma [37], and considering the upper bound for the exponential decaying term $|\pi| \leq \text{Re}^{-\lambda t}$ given in A4, we have $V(t) \leq V(t_0) + c_R e^{-\lambda (t-t_0)}$, for all $t \in [t_0, t_1]$ and $t_1 \in (0, t_0)$, where $c_R > 0$ is an appropriate constant and $0 < \lambda < \min (\lambda, \lambda_{\text{min}})$. Applying the Rayleigh-Ritz inequality, we finally obtain (19).

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